

Philadelphia '83  
Users Days

CHAPWP  
CAPWAP/C

(CAPWAP Analysis Using the Characteristics Approach)  
by Frank Rausche

(In the following description "d" may stand for "delta", i.e., for a finite increment of length or time)

CAPWAP/C is a program that in general works like CAPWAP except that it uses the characteristics method rather than the lumped mass approach for analysis. The characteristics method divides the pile in  $N_p$  segments which are of uniform cross-section. Each segments,  $i$ , has a length,  $dL_i$ , such that its wave travel time,  $dt_i$ , equals the analysis time increment,  $dt$ . Thus, for variable pile properties  $E_i$ ,  $W_i$  (elastic modulus, specific weight), the wave speed of a segment is

$$c_i = (E_i g/W_i)^{1/2}$$

where  $c_i$ ,  $E_i$  and  $W_i$ , may be average properties over a segment's length if the properties change within the corresponding length increment,  $dL_i$ , and  $g$  is the earth gravitational constant.

$$dL_i = (dt)C_i$$

Note that the segments are not of equal length. Resistance forces  $R_k$  may act at the bottom of any segment. They are the sum of the usual elasto-plastic and linearly viscous resistance values. Thus, with a segment velocity,  $v_k$ , and a segment displacement,  $u_k$ ,

$$R_k = R_{sk} + Jv_k$$

with

$$R_{sk} = k_{sk} u_k$$

and

$$R_{nk} \leq R_{sk} \leq R_{uk}$$

The lower static resistance bound is

$$R_{nk} = -U_n R_{sk}$$

with

$$0 \leq U_n \leq 1$$

Note that  $U_n$  is always zero for end bearing and equal to  $R_{sk}$  for skin friction,  $k_{sk}$  is the soil stiffness of the k-th resistance. For positive velocities

$$k_{sk} = R_{uk} / q_{km}$$

with  $q_{km}$  being the modified quake at element k. With,  $q_k$ , the actual quake

$$q_{km} = q_k$$

for  $v_k \geq 0$  and

$$q_{km} = q_k (e_s)$$

for  $v_k < 0$

The factor  $e_s$  may be different for skin and toe and is limited to the range between 0.01 and 1.0.

Soil resistances may act at each pile segment  $i$ . However, since the pile segments are usually small for the characteristics method it may be sufficient to have soil resistance at the bottom element for both end bearing and skin friction. Above that point a constant number of segments may be skipped. Thus,  $N_p$  may be different from  $N_s$ , which is the number of soil resistances. Figure 1 shows a general model.

At any time,  $t_j$ , both upwards and downwards traveling waves,  $F_{u,i,j}$  and  $F_{d,i,j}$  are present in segment,  $i$ . For two neighboring segments of equal properties.

$$F_{u,i,j+1} = F_{u,i+1,j}$$

and

$$F_{d,i} = F_{d,i-1,j}$$

If the cross sectional properties change between  $i$  and  $i+1$  then

$$Z_i = E_i A_i / c_i$$

and

$$Z_{i+1} = E_{i+1} A_{i+1} / c_{i+1}$$

where  $A_i$  is the cross sectional area of segment  $i$  and  $Z_i$  is the segment impedance. Using for simplification

$$Z_{n,i} = Z_i / (Z_i + Z_{i+1})$$

and

$$Z_{s,i-1} = Z_i / (Z_i + Z_{i-1})$$

the new waves are, in general, determined from

$$F_{u,i,j+1} = Z_{r,i} [2F_{u,i+1,j} - F_{d,i,j} + R_i] + Z_{s,i+1} F_{d,i,j}$$

$$F_{d,i,j+1} = Z_{s,i} [2F_{d,i-1,j} - F_{u,i,j} + R_i] + Z_{r,i-1} F_{u,i,j}$$

If compressive or tensile slacks,  $S_{c,i}$  or  $S_{t,i}$  are prescribed then

$$F_{u,i,j} = F_{d,i,j-1} + R_i$$

and

$$F_{d,i+1,j} = -F_{u,i+1,j-1}$$

as long as the segment separation is within the slack zone. (Figure 1b).

Internal damping may be added (although apparently unnecessary for reasons of numerical stability) by computing the change of a wave and reducing the new wave by a specified fraction. Thus,

$$F_{u^*,i,j} = F_{u,i,j} - c_p (F_{u,i,j} - F_{u,i,j-1})$$

$$F_{d^*,i,j} = F_{d,i,j} - c_p (F_{d,i,j} - F_{d,i,j-1})$$

The \* indicates the dampened wave value;  $c_p$  is usually less than 0.02.

At the pile top, either force,  $F_{m,j}$ , or velocity,  $v_{m,j}$ , are prescribed. Then the complementary quantity is either

$$F_{c,j} = Z_1 v_{m,j} + 2F_{u,1,j-1}$$

or

$$v_{c,j} = [F_{m,j} - 2F_{u,1,j-1}]/Z_1$$

At the pile toe

$$F_{u,Np,j} = -F_{d,Np,j-1} + R_{Ns} + R_{Ns+1}$$

with  $R_{Ns+1}$  denoting the toe resistance.

The force at segment  $i$  is

$$P_{i,j} = F_{u,i,j} + F_{d,i,j-1}$$

and the velocity

$$v_{i,j} = [F_{d,i,j-1} - F_{u,i,j}]/Z_i$$

thus displacements become

$$u_{i,j} = u_{i,j-1} + (1/2)(v_{i,j-1} + v_{i,j})dt$$

Several comparison analyses have been performed so far with the original program as a reference.

Preliminary studies have shown that

- (1) The analysis is stable unless the static soil resistance becomes unusually stiff.
- (2) High viscous damping forces do not lead to instability, as the Smith model does.
- (3) If time increments are chosen at an approximate frequency of 6000

samples per second, usual pile driving records are sufficiently accurately represented.

- (4) If soil segments are chosen at every third pile element and at a 6000 sps frequency, then the CAPWAP/C analysis is approximately 20% faster than CAPWAP. Further time savings can be obtained in cases with little or no skin friction over substantial pile portions (offshore).
- (5) The response at time  $2L/c$  is much more accurate than that of the lumped mass analysis, in particular on long piles. Thus model changes to avoid phase shifts are unnecessary.
- (6) Soil mass must be introduced as a change of specific weight over the affected pile bottom portion. Then a complete model change with a new  $dt$  and new segment lengths,  $dL_j$ , has to be computed.
- (7) Because of the requirement for variable time increments the measured force and velocity values have to be determined at the analysis frequency by interpolation between the originally digitized values. Appropriate smoothing is done automatically.
- (8) The analysis is sensitive to sudden changes (discontinuities) of the input force (CHAPWP) or velocity (WAPCHP). Thus the force static offset at time step  $j=1$  must be subtracted and added to all - static or dynamic force results.
- (9) The program checks on the activation time increments,  $j_{x,i}$ , at which the static resistance,  $R_i$ , is activated for the first time. It then plots tickmarks at that time increment,  $j_{a,j}$ , at which the fully activated resistance effect reaches the pile top. Note that

$$j_{a,i} = j_{x,i} + i$$

The following additional features have been or will be added to CAPWAP/C

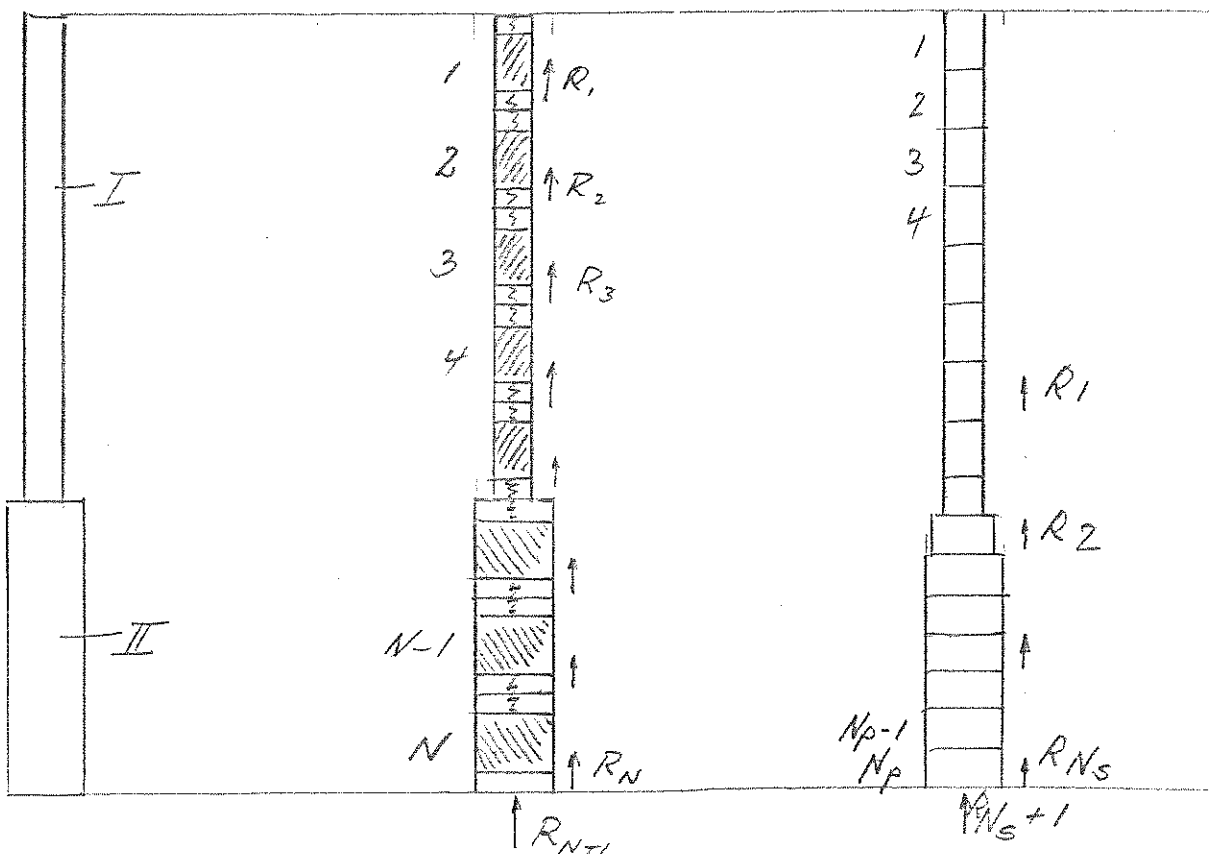
- (1) Residual analysis
- (2) Best result determination, i.e. an error or difference evaluation
- (3) Improvement suggestions based on (2)
- (4) Automatic distribution including effects of unloading. This will be accomplished by consideration of the ratio between wave length and distance between resistance locations.

Figure 1a:  
 Model Comparison  
 (Example 2 material pile)

Real

CAPWAP

CHAPWAP



Elements  
 of Equal  
 Length;  
 Variable  
 Mass,  $m$  and  
 Stiffness,  $k$

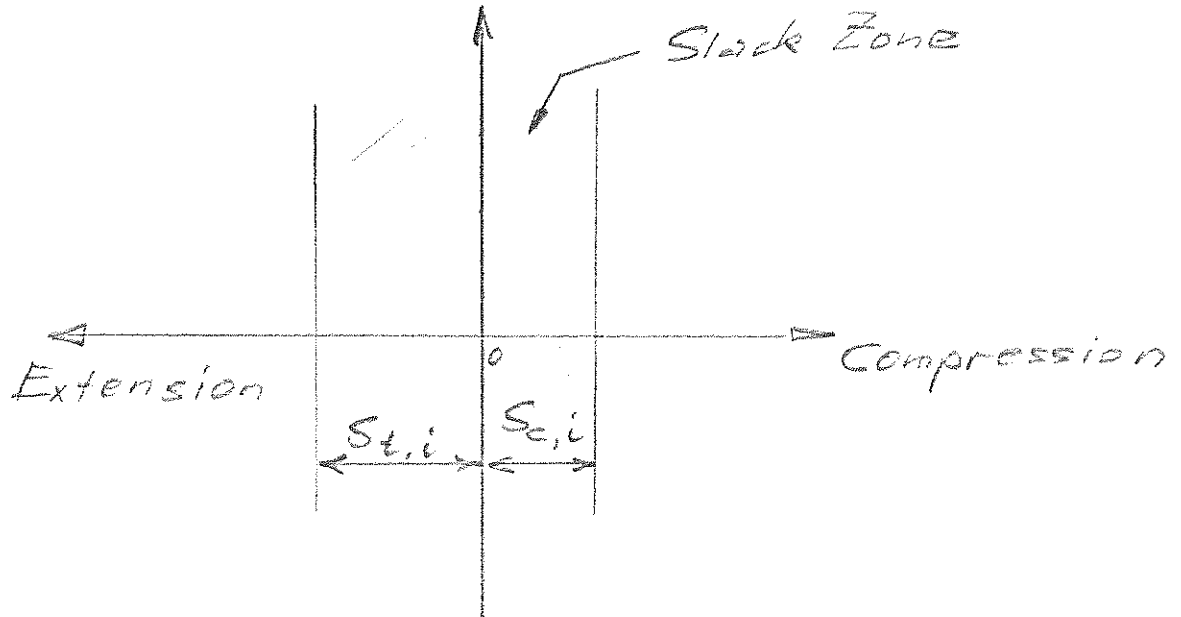
Elements  
 of Equal  
 Travel  
 Time;  
 Variable  
 Impedance

Unequal  
 travel  
 time =  $\sqrt{\frac{m}{k}}$

Unequal  
 Length =  $c \cdot \Delta t$



Figure 1b:



$$\text{Extension} = u_{i+1,j} - u_{i,j}$$

with  $u$  being the segment displacement.

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