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LOAD AND RESISTANCE FACTOR DESIGN
LEVEL II METHODS

INTRODUCTION

Engineers face a great responsibility when they are asked to design any structure, especially if the structure is frequented by many people or must protect the environment from contamination. I would hope that if we could employ unlimited resources that our structures would be as safe and as beautiful as possible. Unfortunately, every design must be a balance of safety, economy, and service so as to fit within the constraints of our resources, time, and values. The way our society has elected to insure this balance is through the development and enforcement of building codes.

A good question to ask is 'What came first, the building code or the design practice?' According to Corotis[1], in general the building codes have been developed from existing practice and method. Acceptable design was that which worked well and with enough successes it would become part of the code. Practice gave birth to code and code changes.

This pattern may soon be broken. Advances in scientific methods, mathematical modeling, data analysis and probability theory may enable codes, calibrated to previous codes, to lead practice. The task is not an easy one nor is it necessarily a popular approach to design.

This report looks briefly at the historical development of a probabilistic approach to structural design and at fundamental reliability theory. The main focus, however, is upon level II methods and how they may be employed to provide specification groups and code writers with the means to develop more intelligent and rational codes, codes that may soon lead practice. A proposed method for determining resistance factors for steel piles is then presented.

HISTORICAL BACKGROUND

When existing codes are examined it becomes apparent that statistical variation and probability distributions are already included in some design values. An example is the acceptance criteria for concrete strength in ACI Standard 318-77. It is designed to insure that the probability of obtaining concrete with a strength less than f'c is less than 10 percent.[2] Other examples of the existing use of probability are the distributions of maximum wind speeds and the annual extreme snow loads in the determination of load effects.

Faced with uncertainties like these and in most every loading condition and material strength, we must accept the fact that a deterministic approach to structural design is not acceptable. Instead, it is paramount that we develop probability theory and reliability-based design so that the risk allowed, whatever the limit-state, is acceptably small.

According to NBS 577[3], "the use of statistical methodologies has stopped at the point where the nominal strength or load was specified. Additional load and resistance factors, or allowable stresses, were then selected subjectively to account for unforeseen unfavorable deviations from the nominal values." By selecting an appropriate level of safety and utilizing reliability methods it is now possible to design structures with a consistent probability of failure throughout based on the inherent uncertainties in its components.

STRUCTURAL RELIABILITY THEORY

At the base of structural reliability theory is classical reliability theory, which has its origins in the determination of the reliability of mechanical and electronic systems. Engineers were attempting to predict such things as the expected down time for a machine or the life expectancy of a particular electronic component. Important considerations such as when maintenance should be scheduled and which parts should be replaced were set within a probabilistic framework. In most of these cases the reliability of the component or system was a function of the operating or exposure time[4].

Structures exist and function for a certain lifetime. Over this lifetime, the components of the structure are subjected to various loads which are not always constant or predetermined. The materials which make up the components of the structure provide the resistance, and this capacity is dependent on many uncertainties in the manufacturing process, modeling methods, and material strengths. In reliability theory, the loads and resistances are assumed to be random variables and at least some statistical information about the variables is known.

Generally there are four levels of reliability methods which are employed in determining the safety of a structure. When considering the performance of a structure, the concept of a limit state must be determined in order to arrive at an acceptable measure of reliability. The two most useful limit states are that of serviceability and the ultimate limit state. Here is a brief look at the four levels.

Level IV is referred to as the stochastic format. Corotis[1] represents the level IV format as:

$$P_S = P\ [L(s,t) < R(s,t)]$$

s throughout the structure.
t over the economic lifetime.
L(s,t): various components of loading.
R(s,t): various components of resistance.

This equation indicates that the probability of safe performance is the probability that each component of the load effect vector is less than the corresponding component of the resistance vector, throughout the space of the structure and during the full design lifetime

The approach is simple in concept but not practical because of the difficulty of handling multi-dimensional stochastic variables. Corotis alludes to even higher level formats which require optimization of design and the performance benefits of the best allocation of resources. These approaches are far too complex and are impractical for specification writing groups.

Level III may be considered the full distribution format. On this level calculations are used to determine the 'exact' probability of failure of a structure. The actual probability density functions of all the random variables which make up the loads and resistances are known. A simplification from level IV is the assumption that the resistance does not vary over time. The probability of failure is the event that the resistance, R, is less than or equal to the load S :

$$\begin{aligned} p(f) &= P(R < S) \\ &= \int_0^\infty [f_R(r)\,dr]\,f_S(s)\,ds \\ &= \int_0^\infty F_R(s)\,f_S(s)\,ds \end{aligned}$$

where $f_R(r)$ and $f_S(s)$ are the probability density functions and $F_R(s)$ is the cumulative distribution function of R.[5]

If both R and S are normally distributed the probability of failure may be defined like so:

$$p(f) = 1 - \Phi\left(\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 - \sigma_S^2}}\right)$$

where μ_R and μ_S are the mean values and σ_R and σ_S are the standard deviations. This can also be expressed very easily for lognormal distributions. The application of level III methods is much more complex than it appears here. Even if it is possible to know the probability densities for certain variables, the joint density probabilities are in almost every case beyond practical reach.

Level II formats are referred to as first-order, second-moment methods. An attempt is made to approximate the failure probability of a structure through iterative calculation procedures. In 1969 Cornell[6] proposed a relatively simple and straightforward approach to creating probability-based structural codes. "It is the coupling of best predictions of strengths, loads, deflections, etc. with quantitative measures of the dispersion or uncertainty in these predictions that remains the fundamental advantage of a probabilistic basis for a code."

The predictions of the resistances and loads, as well as the measures of uncertainty in those predictions are determined upon the statistical data available. A measure of the degree of safety required for a particular limit state is specified and the safety factor necessary can be determined. Level II methods are explored in more detail in the next section.

Level I is considered the load and resistance factor format. Approximate degrees of safety or reliability are provided on an element or component basis. This is accomplished through the use of partial safety factors applied to nominal values of loads and resistances.

The ACI 318 code has utilized partial safety factors, or load and resistance factor design, since 1963. The nominal resistance of an element is multiplied by a specified strength reduction factor with a value less than 1.0 and assumed load effects are multiplied by specific load factors with values greater than one. These factors are adjusted for different combinations of loads and for the importance of a structural member. The nominal values employed are not necessarily the mean values for strength and are in most cases somewhat conservative. A typical design expression appears as such:

$$\phi R > \gamma_d DL + \gamma_l LL$$

where ϕ = the strength reduction factor.
γ_d = the load factor for dead loads.
γ_l = the load factor for live loads.

Although level I formats are not strictly methods of reliability analysis they are methods of safety checking and do provide the base upon which to impose level II methods.

LEVEL II METHODS

When designing it is necessary that the resistance of an element and the effects of the applied loads are defined in some manner. How can we best determine these values given the practical constraints within which we must operate? The determination of the probability density functions for all the random variables is currently too difficult. To use the accepted nominal values works but tells us nothing of the dispersion of the values. Also, the nominal values are based on theoretical models which themselves are imperfect.

Level II methods suggest that we utilize two statistical parameters of the random variables that are readily available and easy to work with. These are the means and the standard deviations or coefficients of variation (c.o.v.). In this manner we can introduce consideration of variability without requiring full probabilistic analysis.

The expected value, $\overline{R}$, of the resistance is a function of the expected values of the material properties and the cross-sectional dimensions. The c.o.v. of $\overline{R}$ can be referred to as V(R) and is equal to:

$$V(R) = \sqrt{V(M)^2 + V(F)^2 + V(P)^2}$$

where V(M), V(F), and V(P) are the material c.o.v., the fabrication c.o.v., and the professional(model) c.o.v. respectively.

The expected values of the loads are multiplied by some constant to reflect the effect of the loads. V(S), the uncertainty associated with the applied loads is equal to:

$$V(S) = \sqrt{V(T)^2 + V(E)^2}$$

where V(T) and V(E) are c.o.v. of the total load and the c.o.v. of the load effect.

The reliability index β is now defined as:

$$\beta = \frac{\overline{R} - \overline{S}}{\sqrt{\sigma_R^2 - \sigma_S^2}}$$

or

$$\beta = \frac{\ln \overline{R}/\overline{S}}{\sqrt{V_R^2 + V_S^2}}$$

Figure 1 displays the probability density function for R - S. The shaded area to the left of the origin is equal to the probability of failure.

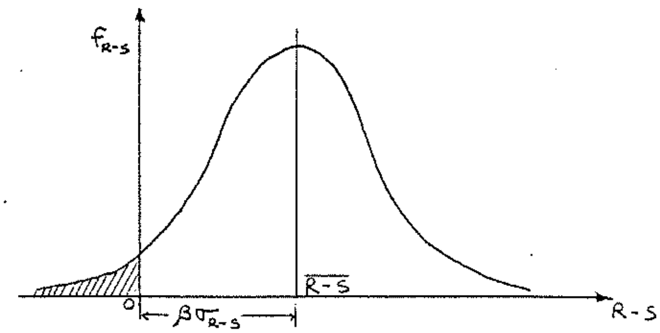

FIGURE 1.

If R and S are normal variates $N(\mu_R, \sigma_R)$ and $N(\mu_S, \sigma_S)$, then the probability of failure may be defined as:

$$p(f) = 1 - \Phi\left[-\left(\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right)\right]$$

$$= \Phi(-\beta)$$

A central safety factor can be defined which reflects the degree of reliability required for a specified limit state or type of performance[6]:

$$\Theta = \frac{1 + \beta\sqrt{V_R^2 + V_S^2 - \beta^2 V_R^2 V_S^2}}{1 - \beta^2 V_R^2}$$

As the value of β, the reliability index, increases, it is easy to recognize that the safety will also increase. Figure 2 displays the central safety factor, Θ, for β = 4.

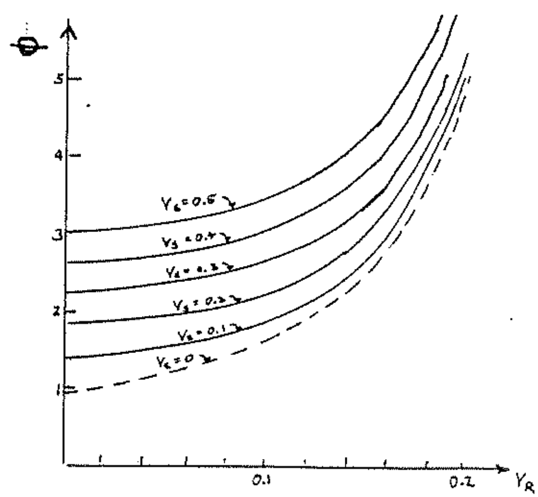

FIGURE 2. [6]

RESISTANCE FACTORS

Resistance factors, also known as strength reduction factors, strength modification factors, capacity reduction factors and other like expressions, are not new to structural engineers in either concept or practice. As mentioned earlier when discussing Level 1 formats the ACI code has employed resistance factors since the early sixties. Although not a probabilistic approach to load and resistance factor design this was an attempt to account for the likelihood of materials with less than expected strengths and the consequences of failure.

The approach of both the ACI and the new AISC code is to determine resistance factors for a particular structural action such as bending or compression. Another method is that of the Comite Euro-International du Beton (CEB) and its Model Code. Partial safety factors are utilized and are the same for all limit states. For the case of reinforced concrete the nominal yield strength of the steel is multiplied by one partial resistance factor and the ultimate compressive strength of the concrete is multiplied by another partial resistance factor. If the member is a beam column, the axial strength is multiplied by an axial strength factor and the bending strength multiplied by a separate resistance factor. An excellent comparison of the two methods is available in NBS 577[3].

Just as the single load factor is not considered acceptable by the ACI code maybe an overall resistance factor has the same shortcomings. Partial resistance factors permit flexibility in the inclusion of variability of distinct aspects of the resistance.

There are four general steps in the calculation of resistance factors:

1. Selection of an algorithm for the nominal resistance of the element.

2. Calculation of the mean and coefficient of variation of the resistance of the element using the relationship of step 1 and the available information on the mechanical properties of the material and test results on the element.

3. The resistance factor is calculated by:

$$\phi = \exp(-\alpha\beta \, V_R) \frac{R_m}{R_n}$$

where

α = 0.55
β = 2.0 - 5.0
Rm = the mean resistance of the member
VR = the coefficient of variation of the resistance

4. In most cases the resistance factor is expressed as a function of a characteristic value and may vary over the range of the variable. Such a variable is the slenderness ratio for columns.

RESISTANCE FACTORS FOR STEEL PILES

A starting point for the determination of resistance factors for steel piles is the work of Galambos and Ravindra in the late 70's.[7] Employing first-order second-moment probabilistic methods, they developed design criteria for steel in a Load and Resistance Factor Design format. I feel that their resistance factors for short columns, and the methods in calculating those factors are a good beginning in determining steel pile resistance factors.

This is not to say that there are not problems with their approach or that we can base pile strength solely on the modeling of columns. First, I would like to present again the method of calculating resistance factors for structural elements in general and columns in particular. Secondly, I will examine the applicability of the method to steel piles and look at the results of Galambos and Ravindra for short columns. I will close by considering the major factors in deriving resistance factors and the special considerations for piles.

Here are the four steps in the calculation of resistance factors:

1. Selection of an algorithm for the nominal resistance of the element. For columns this is:

$$R_n = A_g F_{cr}$$

where A_g = the gross area and
$F_{cr} = F_y(1 - 0.25\lambda^2)$ for $\lambda <= 2$
$F_{cr} = F_y/\lambda^2$ for $\lambda >= 2$
$\lambda = \frac{KL}{r}\frac{1}{\pi}\sqrt{\frac{F_y}{E_s}}$

If we restrict ourselves for now to piles confined over their entire length the effective slenderness ratio may be taken equal to zero because the chance of buckling is almost nonexistent.

2. Calculation of the mean and coefficient of variation of the resistance of the element using the relationship of step 1 and the available information on the mechanical properties of the material and test results on the element.

Galambos and Ravindra examined characteristic and representative sets of data and estimates were made of the mean values and coefficients of variation. They felt that at the time this was this best approach considering the scatter in time and place of the available data and the lack of consistency in control and methods.

As expected, almost all the available data is from tests on wide flange sections and not pipe, box, or H sections.

3. The resistance factor is calculated by:

$$\phi = \exp(-\alpha \beta V_R) \frac{R_m}{R_n}$$

where $\alpha = 0.55$
$\beta = 3.0$
R_m = the mean resistance of a column
$= \overline{\sigma_{cr}/F_y} (\overline{P}\ \overline{M}\ \overline{F})$

where

$\overline{\sigma_{cr}/F_y} = 1.0$ for $\lambda <= 0.15$
R_n = the nominal resistance
V_R = the coefficient of variation of the resistance and may be approximated as:

$$V_R = \sqrt{V_M^2 + V_F^2 + V_P^2}$$

where M = the variability of the material properties
F = the uncertainties of the fabrication process
P = the uncertainties of the professional assumptions

The reliability index β was chosen as 3.0 for beams, columns, and beam columns. Keeping β constant for these elements will hopefully level out variations in reliability and produce a more uniform design. β may be increased or decreased depending upon the desired level of safety for a structure.

Exempting very unusual circumstances the reliability index for steel piles ought to be 3.0 or whatever has been chosen for a structure.

V_F , the fabrication tolerance is assumed to be 5% for standard steel sections. The material property statistics, E and F_y , are extracted from tests and accepted handbook values. Both V_F and V_M for piles should not differ from the values calculated for columns unless data can be found for or generated for H, box, and pipe sections.

The variability associated with the professional or model assumptions for steel piles depends on factors that differ from those associated with columns. The slenderness ratio, which is the characteristic variable for columns, should not be of importance. V_P for piles eventually will reflect our confidence in whatever model is chosen for predicting pile strength.

4. In most cases the resistance factor is expressed as a function of a characteristic value and may vary over the range of the variable. Such a variable is the slenderness ratio for columns. At this point, no such characteristic variable has been determined for piles.

Galambos and Ravindra utilized the following resistance statistics in developing resistance factors for steel columns:

λ	= 0.3	0.5	0.9	1.1
$\overline{\sigma_{cr}/F_y}$	= 0.936	0.849	0.646	0.539

$\overline{P} = 1.03$ $\qquad V_P = 0.05$
$\overline{M} = 1.05$ $\qquad V_M = 0.05$
$\overline{F} = 1.0$ $\qquad V_F = 0.05$

$\overline{P}\overline{M}\overline{F} = 1.08$ $\qquad V_R = 0.12$

As λ increases the only changes required to compute the resistance factor are for $\overline{\sigma_{cr}/F_y}$. Again, this is because of the importance of the slenderness ratio for columns but is not of importance for axially loaded piles.

The results are:

$$\phi = 0.86 \quad \text{for} \quad \lambda <= 0.16$$
$$\phi = 0.90 - 0.25\lambda \quad \text{for} \quad 0.16 <= \lambda >= 1.0$$
$$\phi = 0.65 \quad \text{for} \quad \lambda >= 1.0$$

So, for a reliability index, β, of 3.0 and a slenderness ratio less than 0.16 a resistance factor for steel H piles may be taken as 0.86.

Let's look back at the major factors considered in deriving these resistance factors.

1) The variability in member strength due to variability of material properties. If we accept the test data reported for wide flange sections as applicable to pile sections the statistics reported by Galambos should be used.

2) The variability in member strength due to variability of dimensions. The fabrication of hot rolled steel sections is well controlled and it appears reasonable to assume a coefficient of variation of 5%.

3) The variability in member strength due to simplifying assumptions in the resistance equations. Here is where the differences between the expected strengths of piles and columns will affect the resistance factors. The confidence in the model chosen for piles will determine the magnitude of the variability. The effects of pile driving on the capacity of the pile must also be taken into account when determining resistance factors.

4) The importance of member in structure. An example of this is the different resistance factors assigned to columns and beams in the ACI code. Also the importance of the type of structure and the consequences of failure need to be considered in the selection of the reliability index, .

5) The designers' familiarity with the method used.

CONCLUSIONS

It appears as though the level II methods are very powerful and yet relatively easy to apply. By obtaining readily available statistical parameters, such as the means and variances of random variables, probability-based code is possible. The individual uncertainties inherent in the resistances and the applied loads can be studied individually and combined in a specified method consistent with a desired level of safety.

Level II methods are not without fault. One difficulty is that the limit-state equation which relates the resistance and load variables is linearized at the mean values. When that equation is nonlinear and higher order terms are not considered, unacceptable error results. Another problem concerns the the failure of level II methods to be invariant to different equivalent formulations of the same problem. This is called a lack of function invariance. The result is that the reliability index depends on how the limit state is formulated.

Both of the difficulties noted above may be overcome by employing more advanced level II methods.

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