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Wave Mechanics and the Wave Equation

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ABSTRACT

The following paper summarizes ideas and concepts that may be helpful to the practicing engineer or contractor w1ithout getting into the details about complicated derivations. Even though wave mechanics is the topic, it was felt that a review of energy concepts may also be important for an understanding of the mechanics of pile driving. Similarly, a brief description of both the propagation of stress waves and the wave equation model will be helpful for an understanding on how these mathematical tools can be simply and effectively used to solve practical problems.

ENERGY CONSIDERATIONS

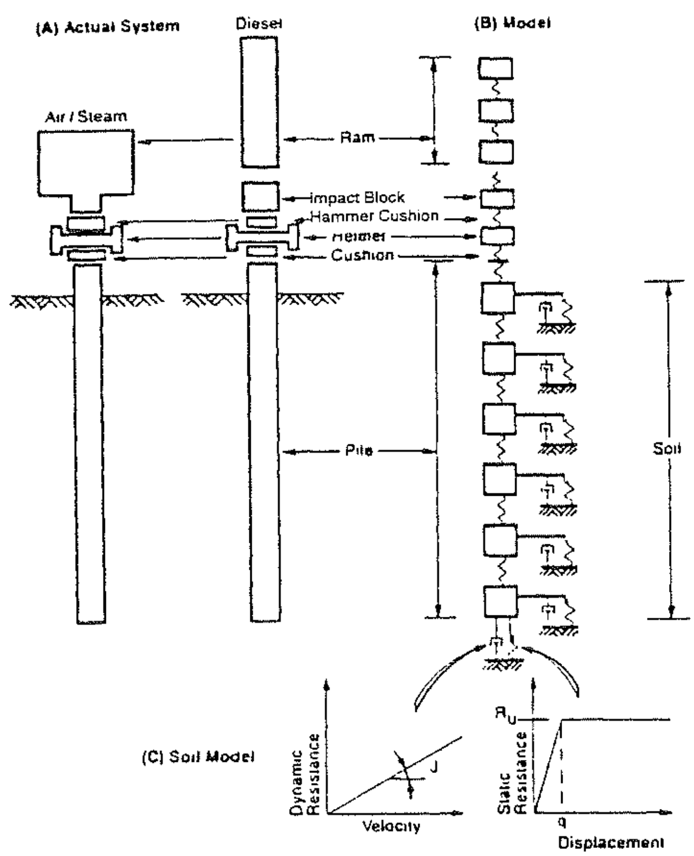


Figure 1: Hammer, driving system, pile and soil, schematics and wave equation model

(Reference is made to Figure 1, left side, which shows schematics of two different types of hammers and the associated wave equation models which will be discussed below.)

Hammer energy

Energy is a measure of the amount of work that can be done: push a mass over a rough surface, lift a weight, compress a volume of gas, accelerate a mass to a certain speed, compress a spring, etc. Mathematically energy is defined as the product of force and distance over which the force acts. For example, the weight of a ram, W_R, suspended a distance, h, above a reference datum, has the ability to do work equal to its potential energy

$$E_P = W_R\, h \qquad (1)$$

Since energy is "indestructible" (in the worst case it is converted to heat, sound or other forms of energy), after falling through a vertical distance h, the ram now contains a kinetic energy, E_K while its potential energy has become zero.

$$E_K = ½ v_i^2 W_R / g \quad (2)$$

where g is the gravitational acceleration and v_i is the velocity of the ram just before impact. Therefore, with $E_K = E_P$,

$$v_i = \sqrt{(2gh)} \quad (3)$$

This is the ram impact velocity in the absence of any friction or other losses. The work that a friction force, R, would be doing while the ram is falling is

$$E_F = R\, h \quad (4)$$

practically reducing the available energy from W_R h to $(W_R - R)h$. Let us assume that the friction force is a certain percentage, μ, of W_R, then we can write the reduced potential energy as

$$E_{PR} = (W_R - R)h = (W_R - \mu W_R)h = \eta W_R\, h \quad (5)$$

Where $\eta = (1 - \mu)$ is the so-called efficiency of the hammer while μ indicates the fraction of the energy that has been converted to heat rather than speed. Of course, the impact velocity of the ram is now

$$v_i = \sqrt{(2gh\eta)} \quad (6)$$

Diesel hammer compression energy

In addition, to friction, other losses also occur during the descent of a hammer. For example, a diesel hammer compresses air which requires the following energy:

$$E_D = \int (p\, dV) = A \int (p_{IN} [V_{IN} / V]^{exp})ds$$
$$= [A\, p_{IN} /(exp - 1)]\, [V_{IN}/A + h_C]^{exp}\, \Lambda \quad (7)$$

with

$$\Lambda = 1 / [V_{IN}/A]^{(exp - 1)} - 1 / [h_C + V_C/A]^{(exp - 1)} \quad (8)$$

In this formula, V_{IN} is the initial volume of air that is present in the diesel hammer chamber when compression starts, A is the inside area of the cylinder, p_{IN} is the initial pressure (atmospheric), s is the distance that the ram has traveled after the compression started, V is the associated volume, h_C is the compressive stroke, and exp is the exponent of adiabatic compression, typically 1.4 for air. In the wave equation program GRLWEAP exp is 1.35. The chamber volume, V_C, i.e. the volume left over during impact, plus the product of compressive stroke h_C and area A equal the initial volume. The compression ratio is therefore V_{IN} / V_C. Note that the atmospheric pressure does also do positive work on the top surface of the ram. This energy is merely the product of atmospheric pressure times ram top area times compressive stroke.

As an example, let us use a few realistic numbers representing a typical diesel hammer with 2 ton ram mass. Assume h_C = 15.5 inches, V_C = 158 $inch^3$, A = 124 $inch^2$, exp = 1.35, atmospheric pressure of 14.7 psi. Then V_{IN} = 158 + (15.5)(124) = 2080 $inch^3$ and the compression ratio is 13.2. Entering these numbers in the above equation and considering the work of the atmospheric pressure yields a pre-compression energy of 8.4 kip-ft.

If this hammer has indeed a ram weight of 2 tons and if its maximum stroke is 10 ft, then it would be rated with 40 kip-ft. The energy stored in the air during pre-compression is therefore 21% of the rated energy.

Energy in the Driving System

During impact the ram also compresses one or two cushions. If the spring constant of the cushion is k and the maximum compression x and the maximum force F = k x, then the energy stored in the cushion at full compression is

$$E_C = \frac{1}{2} F x \qquad (9)$$

Let us compare a 1 inch thick plywood cushion with a 10 inch thick plywood cushion, both with a cross sectional area of 100 $inch^2$ and normal plywood modulus (30 ksi). The corresponding cushion stiffness values are k = (100)(30) / 1 = 3,000 kips/inch and 300 kips/inch, respectively. A mass falling onto these springs and producing a force of 300 kips in the pile underneath will produce the following respective cushion compression values

$$x = 300/k = 0.1 \text{ and } 1 \text{ inch}$$

and therefore store in the cushion the following energy:

$$E_C = \tfrac{1}{2}\, 300\ (0.1) \text{ or } \tfrac{1}{2}\, 300\ (1) = 15 \text{ or } 150 \text{ kip-inch} = 1.25 \text{ or } 12.5 \text{ kip-ft}$$

(Actually, the force underneath the softer cushion is probably lower than the force under the thinner and stiffer cushion; this would make the effect, demonstrated here, a little less extreme.) For a concrete pile and cushion of this size it would be reasonable to assume that the hammer rated energy ranges between 25 and 50 kip-ft. The thick cushion therefore stores a considerable amount of hammer energy during its compression phase. The thinner and therefore stiffer cushion, on the other hand, generates rather insignificant energy losses. Assuming a coefficient of restitution of 0.5, which is reasonable for a plywood cushion, half of the compression energy would be converted into heat. The other half is either released to the pile or returned to the ram during its upward movement.

Hammer cushions are subjected to higher forces than pile cushions because of the helmet's high mass which causes high inertia forces. The helmet acceleration maybe 1,000 g's and the helmet weight of the pile with 100 $inch^2$ area may be 2 kips. The inertia force is then 2,000 kips. The stiffness of this cushion may be 100(2,000)/4 = 50,000 kips/inch (assuming a thickness of 4 inches, and a modulus, for wood with grain parallel to load, of 2,000 ksi). The compression x is therefore 2,000/50,000 = 0.04 inch and the energy stored is ½ (2000)(.04) = 40 kip-inch or 3.3 kip-ft. One half of this energy may be converted to heat if the coefficient of restitution is 0.5 which is commonly assumed for wood.

Let us now calculate how much energy it takes to accelerate the helmet to its highest velocity which is approximately equal to the maximum velocity that we see at the pile top, say 10 ft/s. Again assuming a helmet weight of 2 kips yields a helmet kinetic energy of

$$E_{HK} = \frac{1}{2}\, m\, v_i^2 = \frac{1}{2}\, (2/32.2)(10)^2 = 3.1 \text{ kip-ft}$$

However, it is reasonable to assume that this kinetic energy is still doing useful work on the pile after the helmet has started to slow down.

Calculation of energy transferred to pile top

In summary, our hammer may have a rated energy of, say,	40 kip-ft.
Considering a standard efficiency of 0.67, friction losses of a single acting air/ steam hammer amount to (1 - 0.67)(40)	13.2 kip-ft.
Total remaining at impact for the air/steam hammer	26.8 kip-ft
For a diesel hammer with 0.8 hammer efficiency we would normally expect friction losses of 20% or	8 kip-ft.
Also, if it is a diesel hammer, it stores in the compressed gases roughly	8 kip-ft.

(For the diesel hammer there would also be kinetic energy temporarily stored in the impact block. The magnitude of this loss may be comparable to the energy stored in the helmet when it moves. However, most of this kinetic energy is probably transferred to the pile at a time when it still can do useful work on pile and soil.)

Total energy available at impact for the diesel hammer	24 kip-ft.
The hammer cushion stores approximately	3.3 kip-ft.
The helmet kinetic energy is available to do work on the pile, loss	0 kip-ft.
A medium thick plywood cushion would store an additional	6.6 kip-ft.
Remaining energy at the top of the pile for	
Air/steam hammer	16.6 kip-ft or 42%,
Diesel hammer	14 kip-ft or 35%.

The above percentages (energy at top of pile relative to hammer rated energy) are often referred to as transfer efficiencies, transfer ratios, global efficiencies or system efficiencies. Note that a steel pile could receive (6.6/40)100 = 17% more energy than the concrete pile. These findings are in good agreement with measurement results.

Energy losses are generally lower for hammers with low impact velocity, obviously because cushion and inertia forces would be lower and thus the associated cushion strain energy and mass kinetic energy losses. However, experience, measurements and wave equation analyses all indicate that high velocity hammers do rather well when driving gets hard.

WAVE MECHANICS

Mathematics can prove that the following, intuitively reasonable, statements are indeed true for a slender (long relative to its diameter or width), elastic rod whose top is suddenly struck by a mass.

- A suddenly applied force at the pile top will travel down the pile at a wave speed c, which is somewhere between 12,000 (concrete) and 17,000 (steel) ft/s.
- The stress wave will require a time L/c to reach the pile toe (e.g. 5 ms for a steel pile of length L = 85 ft.) Only then will the pile toe begin to penetrate into the soil.
- Depending on the resistance that the pile experiences, a tension or compression reflection wave will be generated at pile toe. It arrives at the pile top at time 2L/c after impact (10 ms after impact for the 85 ft long steel pile).

Impact velocity and ram mass define the energy available at impact. Considering a pile that is directly struck by a ram (no cushion or helmet mass between ram and pile top), in the first instant of contact, the top surface particles of the pile will assume the velocity, v_i, of the ram. Based on wave theory, the corresponding force acting against the falling ram and against the still motionless lower pile particles can be calculated from

$$F = v_i Z \tag{10}$$

where Z is the pile impedance Z given by

$$Z = E A / c \tag{11}$$

with E being the pile's elastic modulus, A its cross sectional area and c its wave speed, given by

$$c = \sqrt{(E / \rho)} \tag{12}$$

where ρ is the mass density of the pile material.

The presence of a cushion reduces the peak force over time and spreads it over a greater time. A helmet mass has a similar cushioning effect (Rausche et al., 1972). Even though cushions and helmet tend to reduce the maximum force at the time of impact, its magnitude is primarily dependent on the impact velocity.

Of course, the force exerted by the pile top against the falling ram slows the ram down. The greater the mass of the hammer the greater its ability to maintain a high force over a long period of time. Therefore, a small hammer will only generate a short force pulse which may be ineffective in maintaining a sustained downward pile movement and therefore pile penetration.

As stated earlier, piles are relatively long, slender elastic bodies whose dynamic behavior is governed by wave effects. In other words, in the first instance after the ram has impacted, only the top of the pile moves downward. It takes a certain time, the wave travel time L/c, before other pile particles, further down the pile, start to move. At the pile toe, a reflection occurs that depends on the resistance that the soil material at the pile toe offers. The following two situations are of particular interest:

- If there is no resistance at all along the pile and at the pile toe, then the pile bottom will move twice the distance that the top moved during impact and the pile toe will then pull the upper pile particles downward. This creates a tension force and an upwaard traveling tension wave which is potentially damaging to concrete piles.

- If there is no resistance along the pile shaft but an extremely high resistance at the toe that prevents any movement of the pile bottom, then the force at the pile toe will be twice the impact force at the pile top. Theoretically, therefore, the hammer can overcome a force twice the impact force of Eq. 10. In practice, because of the need for all materials to compress before they can exhibit resistance, the limit of the soil resistance is about 1.4 times the impact force for piles with predominant end bearing and 1.0 for shaft resistance.

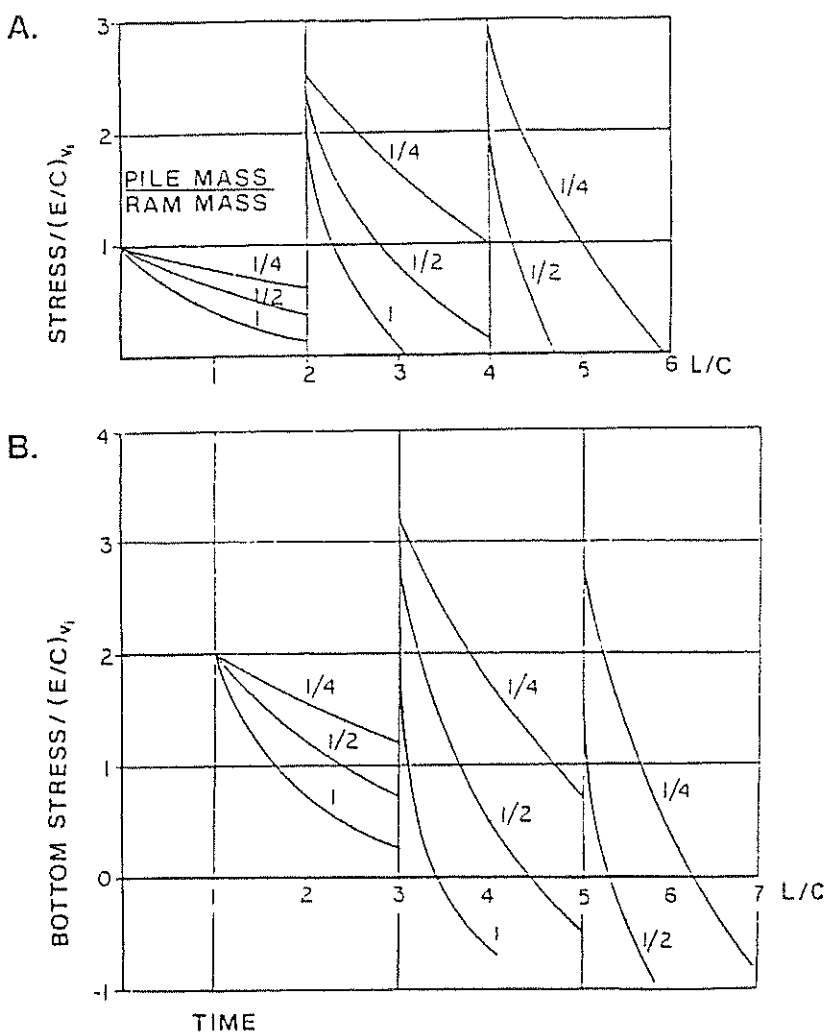


Figure 2: Stresses calculated for pile top (A) and pile bottom (B) by St. Venant using a closed form solution to the wave equation.

Figure 2 shows two non-dimensionalized plots of pile stresses, A at the pile top and B at the pile bottom. They were calculated by St Venant in the 18th century using a closed form solution to the wave equation. Non-dimensionalization was achieved by division with the impact stress at the first moment of ram-pile contact. This solution assumes that a rigid mass impacts the elastic pile top directly and that the pile toe is supported by a rigid foundation. Three curves are shown in each figure, distinguished by different ratios of pile weight to ram weight. The bottom stresses are zero until time L/c when they suddenly increase to twice the stress at impact at the top. Note the slower decay of both pile top and pile bottom stresses for heavier rams which therefore causes higher stresses when the reflection from the pile toe reaches the top as explained in the following.

In the case of the high toe resistance a strong compressive wave reflection occurs which pushes the upper pile particles upwards. For large ram weights, the ram will still have downward momentum when the reflected wave reaches the pile top. In the case of the hard driving pile it is possible that another compressive reflection occurs at the pile top which may cause the pile top force to grow above its initial peak. (Indeed in Figure 2 the pile top stresses reach approximately 2.5 times the impact value at time 2L/c for the case where the ram is 4 times heavier than the pile. Furthermore, the heavier ram will also cause another compressive wave reflection to occur at the pile top producing additional downward pile motions. The benefit of a large ram is therefore not apparent at the time of impact. Instead it generates a longer lasting downward motion of the pile, a further increase of the pile toe force and the chance for additional permanent pile penetration into the ground.

Since it is uneconomical to use very large ram weights, we normally do not see cases where the soil resistance that can be overcome by the hammer is much greater than twice the impact force. However, damage at the pile toe, when it is on rock can easily happen when either the impact force is very high or the ram mass is large. This is seen by inspection of Figure 2 B. The toe stresses first reached 2 times the impact stress at time L/c, i.e. when the impact wave arrives the first time at the pile toe. When the stress wave arrives a second time, bottom stresses exceed a factor 3 for the heaviest ram.

WAVE EQUATION ANALYSIS

Calculations of pile penetrations into the ground and stresses along the pile based on the closed form solutions of St. Venant, yield inaccurate results because of the requirement of very limiting simplifying assumptions. Much more realistic results can be obtained by means of a wave equation computer program, such as GRLWEAP (GRL, 1999) which includes detailed models of hammer, driving system, pile and soil. These programs use a numerical procedure developed originally in the 1950's (Smith, 1960), when digital computers made a discrete solution of wave propagation practical. Smith proposed both an algorithm and associated model parameters. This work is of general interest since it was one of the very first applications of the digital computer to nonmilitary engineering problems. In the United States, computer programs based on this numerical solution became known as the WAVE EQUATION. In the decades following, Smith's concept was evaluated by many researchers and some modifications and improvements were suggested (Goble, et al., 1976; Holloway et al., 1978).

In today's practice, the most commonly used wave equation computer program is GRLWEAP which grew out of WEAP which was originally written under Federal Highway Administration sponsorship. In a wave equation analysis the entire driving system is modeled as a series of masses and springs. The size of the individual mass elements and the stiffness of the springs reflect the mass and stiffness of various components of the real system. The soil is represented by a series of elasto-plastic springs and linear viscous dashpots. A schematic of the entire system model is presented in Figure 1.

Wave equation analyses can answer one or both of the following questions.

- Prior to pile driving: Can the pile be safely driven to the required capacity, given a complete description of pile, soil, hammer, and cushion properties?

- During or after pile installation: What is the static bearing capacity of the pile, given observations recorded during pile driving?

An analysis to answer the first question is generally known as a driveability study. The soil profile is investigated and a pile capacity is computed from soil strength parameters for one or more pile penetrations. Pile type, length, cross sectional area, and materials are selected and a preliminary choice of a suitable hammer is made. Analysis is performed to evaluate the ability of the hammer and driving system to efficiently drive the pile to the required capacity without imposing damaging stresses.

In the second case, an analysis is performed for a pile that is being driven or has been installed earlier and is now being tested during redrive. With the hammer, driving system, pile, and soil parameters all known or estimated, a wave equation analysis is performed for a series of static capacities. The resulting curve relating capacity to blow count is generally called a bearing graph. For any field observed blow count, a pile capacity can then be determined. Typical wave equation analysis results are illustrated graphically in Figure 3, including not only the bearing graph but also stress maxima and, for diesel hammers, the calculated stroke, all as a function of blow count.

The success of a wave equation analysis is dependent, to a large degree, on the realistic model representation of the various components of the pile driving system that generate, transmit, or dissipate the energy of a hammer blow. Modeling of the pile-driving hammer, cushion, pile and soil will be presented here with reference to the GRLWEAP program.

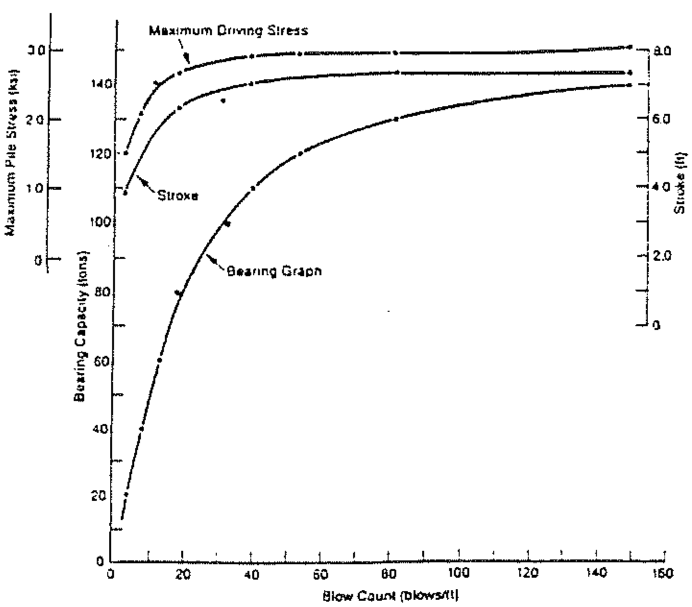


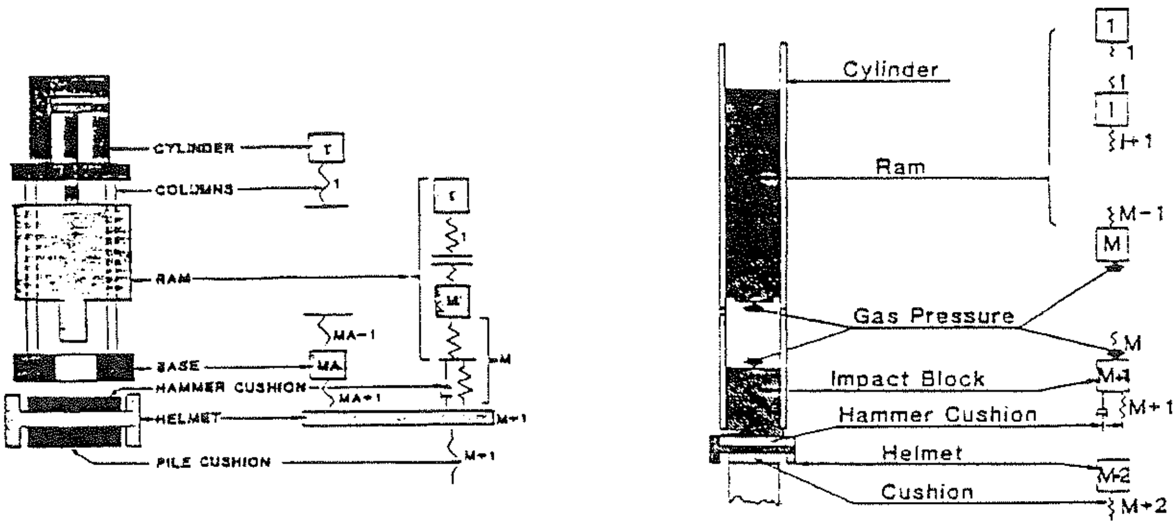
Figure 3: Typical Wave Equation Results.

The earliest and simplest hammers were cable hoisted drop weights. These drop hammers have generally been replaced by hammers using steam, compressed air, or hydraulic fluid to lift the ram. Since all rely on a power source outside the hammer itself, they are often called External Combustion (EC) hammers. By contrast the other common hammer type relies on combustion of diesel fuel inside the hammer to raise the ram for the next blow; these are commonly called diesel or Internal Combustion (IC) hammers. Furthermore, both EC and IC hammers may be subdivided into single, or double acting hammers indicating whether power is supplied to the ram only during the upstroke or also during the down stroke.

For wave equation analyses, masses and springs represent the major components of a hammer model, as illustrated in Figure 4.

The ram of most EC hammers is usually modeled by one or more masses, depending on the length of the ram. GRLWEAP also models the hammer assembly consisting of a cylinder, columns, and base. These components add substantial weight to the driving system and since it collides with the helmet upon pile rebound, it may also have substantial influence on the pile. The relatively slender rams of diesel hammers, are usually modeled with 3 or more segments. Diesel hammers also include an impact block, between ram and cushion, which is represented by an additional mass and stiffness.

GRLWEAP also includes the effect of the diesel hammer pressure, acting between ram and impact block, by thermodynamic analysis. Two different fuel injection types, Liquid (LI) and Atomized (AI), are represented. Basically, liquid injection hammers are those that generate fuel atomization by the impact of the hammer itself. The fuel therefore starts to burn only a short time after impact and this is included in the pressure calculation of GRLWEAP's LI model. Figure 5 shows the pressure-time relationship for modeling the LI process (top) and a comparison between actual and GRLWEAP calculated combustion pressure of a diesel hammer (bottom).



(A) External Combustion Hammer

(B) Internal Combustion Hammer

Figure 4: Wave equation hammer modeling: left schematic of hammer, right spring mass model

In contrast to LI which is started when impact happens, AI combustion starts before impact when a critical pressure is generated or volume formed by the descending ram. At that time, high pressure injection occurs which atomizes the fuel, causing it to burn immediately. As part of the GRLWEAP data file, the models of most common hammers have been compiled and stored for quick recall by the program user. For diesel hammers, the GRLWEAP program computes the stroke (mainly as a function of soil resistance), or a specific (perhaps observed) stroke may be analyzed.

The combination of the hammer cushion, helmet, and pile top cushion is usually referred to as a driving system. These components are designed to protect both the hammer and the pile from serious damage. The hammer cushion is often micarta or nylon. The helmet is a massive steel box that may contain inserts to adapt to certain pile types or sizes. The pile cushion is required when driving concrete piles and usually consists of sheets of plywood. The wave equation modeling of the driving system components considers the helmet as a rigid mass, which is assigned the total weight of the system. Pile cushions are represented as springs with corresponding stiffness computed by the program from each material's elastic modulus E, area A, and thickness t.

The energy dissipation characteristic of all cushions is modeled by assigning a coefficient of restitution. Particularly with pile cushions, both the elastic modulus and the thickness may vary considerably during the driving of a single pile. The pile itself can be modeled very accurately. Whether uniform or not in both geometry and material, the pile is divided into several segments each with a length of about 3.3 ft (1 m). Each of these segments consists of both a rigid mass and an elastic spring whose properties are computed from area, elastic modulus, and mass density. Additional modeling features are also possible, such as a dashpot between the pile segments to account for internal pile damping, or nonlinear springs to model splices or slacks. dependent resistance represents the static soil behavior and is assumed to increase linearly with pile displacement up to a limiting deformation value commonly called the quake, q. Thereafter, continued deformation requires no additional force. Smith originally suggested that a quake value of 0.1 inch be assigned to soil elements both along the pile shaft and below the toe. However, toe quakes up to ten times the value suggested by Smith have been observed (Likins, 1983). An unusually high quake at the pile toe may have a drastic effect on the magnitude of the calculated tension stresses and computed blow counts. The velocity dependent resistance models the soil damping characteristics. The relationship between dynamic resistance and velocity is assumed to be linear. A damping factor, J, defines the magnitude of the calculated damping. The soil's grain size provides a guideline for choosing damping factors along the shaft. High damping factors may limit the pile driveability. Unfortunately, the conditions of high damping or quake usually cannot be foreseen from the subsurface investigation alone.

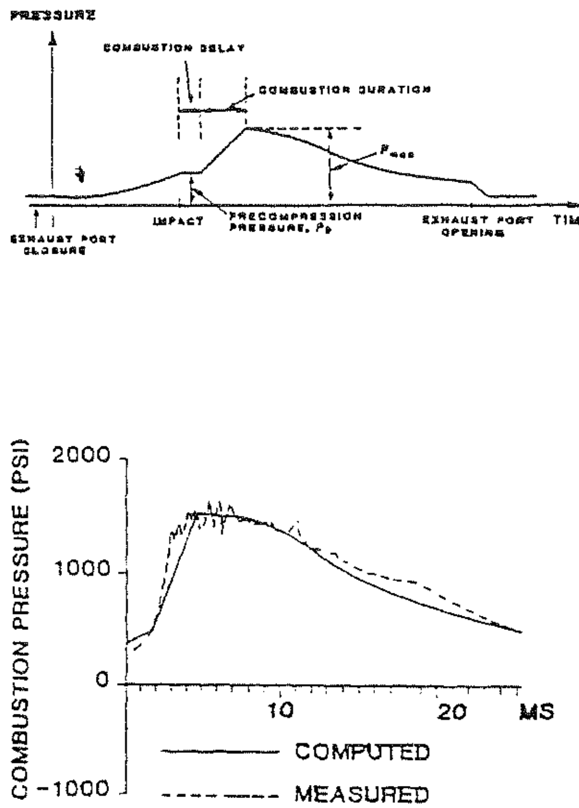


Figure 5: LI pressure time relationship (top) and measured and computed pressures (bottom)

The following is a general summary of the different parameters needed for the execution of a wave equation analysis:

- Hammer: Model and efficiency
- Hammer Cushion: Elastic modulus, area, thickness, and coefficient of restitution.
- Helmet (pile cap): Weight, including all cushion materials and inserts.
- Pile Cushion: Elastic modulus, area, thickness, and coefficient of restitution.
- Pile: Area, elastic modulus, and mass density, all as a function of pile length.
- Soil: Total static capacity, percent skin friction and its distribution, quake and damping values both along the skin and at the pile toe.

In the beginning of the analysis, all pile, soil, and driving system components are assumed to be in a zero stress condition (although the capability to analyze multiple blows for residual stress analyses is available in the GRLWEAP program). A small time step is assigned approximately to one half the travel time of the stress wave through the length of one pile segment. Initially for simple EC hammers, the dynamic analysis begins by calculating the ram impact velocity according to Equation 6.

For the proper choice of hammer efficiency, experience and engineering judgement are required. Without electronic field measurements or familiarity with past performance history of a particular hammer under similar conditions, hammer efficiency is a difficult quantity to estimate. Extensive studies have been directed towards establishing realistic efficiency values for the average behavior of different hammer types (Rausche, et al., 1986). Today the following values have been suggested

- all diesel hammers 80%,
- single acting EC hammers 67%,
- double acting EC hammers 50%,
- hydraulic freefall hammers and monitored hydraulic hammers, 95%.

For diesel hammers, the process is similar except that the analysis starts when the downward traveling ram closes the exhaust ports, thereby beginning the pre-compression cycle, and finishes when the ram passes the ports again on its upward travel. The ram velocity is reduced by the efficiency effect immediately prior to impact.

During a time step, the ram moves a short distance, compressing the hammer cushion spring. The force in this spring is computed from its stiffness and deformation, i.e. the distance between the masses attached to the spring. The force thus calculated above a mass together with the force predicted from the previous time step allows for the calculation of the acceleration of the helmet mass using Newton's Second Law. The acceleration acting during the short time step is integrated to yield a change in velocity and displacement of the mass element. Similar computations are made for each pile mass. Some pile segments may also be subjected to a soil resistance force computed from the current pile element velocity and displacement. This soil resistance force would be included in the force equilibrium equation for computing the acceleration of that pile segment. From accelerations, velocities and displacements can be calculated by simple time integration and once this has been done for all segments, the analysis repeats for the next time step. From this procedure, the forces, accelerations, velocities and displacements are computed for each element as a function of time. Once a sufficiently long time period has been analyzed, the pile starts to rebound and the permanent set for the blow is obtained by subtracting an average quake value from the maximum computed pile toe displacement.

For the purpose of a general understanding of the analysis procedure, this description is, perhaps, adequate. Actually, the computational algorithms are much more complex

As already mentioned, for an assumed input capacity, this analysis yields a computed blow count, a maximum compressive stress, a maximum tension stress and, for diesel hammers, a calculated stroke (determined iteratively). Additional, the transferred energy can be calculated for a comparison with measured quantities. If, as is common, several different capacities are analyzed, capacity and stress extrema can be plotted as a function of blow count in a bearing graph. For driveability analyses the calculated quantities are plotted as a function of depth.

Even though GRLWEAP calculations are quite realistic, the actual conditions on a site, particularly the state of maintenance of hammer and driving system and, of course, the soil conditions, usually cannot be predicted with certainty. It is therefore recommended to complement the calculations with field measurements by the Pile Driving Analyzer®. In this way, hammer performance, pile stresses, pile integrity and bearing capacity can be checked. In addition, for an accurate assessment of pile bottom stresses it is necessary to analyze the field measure quantities using CAPWAP® which uses the pile top measurements of the PDA instead of an assumed hammer model to calculate stresses along the pile and the soil resistance including its distribution along the pile. Obviously, PDA and CAPWAP are only helpful during or after pile installation.

EXAMPLES

Example 1: GRLWEAP analysis of H-pile to rock

The following example demonstrates calculated stress results. It was assumed that a Vulcan 012 has to drive an HP 12x53 through soft soils into rock. The pile to ram weight ratio is (.053)(80)/12 = 1/2.8 and therefore rather low (heavy ram). Figure 6 shows the pile top force and the pile bottom force, calculated by the GRLWEAP program. The impact force at the pile top is approximately 400 kips. After time L/c the pile bottom force sharply increases to a value that is approximately 25% higher than the impact force at the top. At time 2L/c after impact the pile top force sharply increases, this time because of a superposition of the upward traveling compression wave from the pile bottom with the downward compression from the still downward moving ram. The maximum pile top force is approximately 50% higher than the impact force. In this example, the maximum stress at the pile top and bottom are 39.6 and 32.5 ksi, while the impact stress is only 26 ksi. Note that bending due to poor hammer-pile alignment at the top or due to a non-uniform rock surface at the bottom, could add substantial additional stresses. The V 012 could drive this H-pile to an ultimate capacity of almost 500 kips, however, it would be necessary that the steel has a higher than A36 strength. Such a high capacity can only be achieved with this pile type, if the hammer has sufficient energy and if the soil resistance is concentrated at the rock and the rock has a high stiffness and there is no rock relaxation. Of course, should soil set-up occur along the pile shaft then even higher capacities could be demonstrated after some waiting time following pile installation.

Example 2: PDA testing of pipe pile to rock

Another example demonstrates with measurements from the PDA a very similar situation. Force and velocity measured near the pile top are shown in Figure 7. In this case a 12x.23 inch pipe of

77 ft length was driven with a Vulcan 06 hammer into rock. The pile to ram weight ratio was also 1/2.8 like in Example 1. In this case the impact force and the maximum force at the pile top were 190 and 300 kips, a magnification of almost 1.6 while the force at the bottom reached 270 kips or 1.4 times the impact force, according to the PDA calculation. The activated capacity was nearly 300 kips. Maximum stresses of 36 ksi occurred again at the pile top

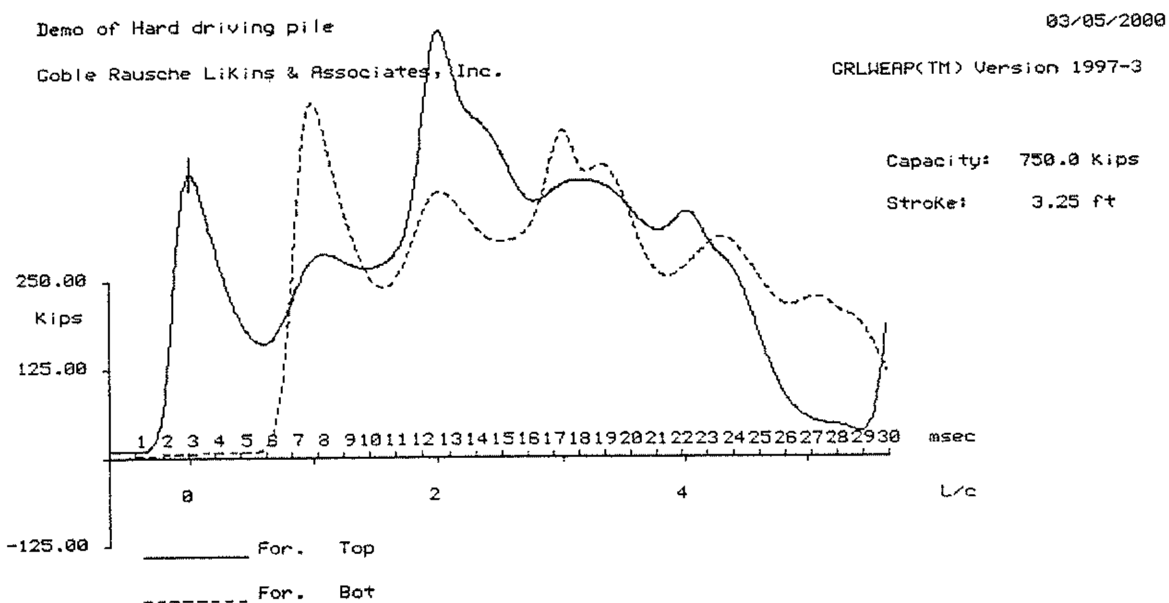


Figure 6: Calculated force at pile top and bottom for a Vulvan 012 driving an 12 HP 53 to rock

CONCLUSIONS

Driving piles to high capacities requires both high energy and impact velocity. Depending on the hammer type, energy may have a different definition and it is recommended that wave equation analyses be performed to select the optimal driving system.

Closed form solutions provide insight in the mechanics of pile driving, the wave equation analysis on the other hand helps obtaining meaningful quantitative results, prior to actually starting the pile driving. Measurements are the best means of assuring that the calculations were done with correct input parameters. When high capacity piles are driven, stresses in the piles will, by definition, be high and to avoid damage accurate measurements of stresses should be taken. To assure that the high capacity is indeed achieved, pile integrity, hammer performance and bearing capacity must be addressed in the testing.

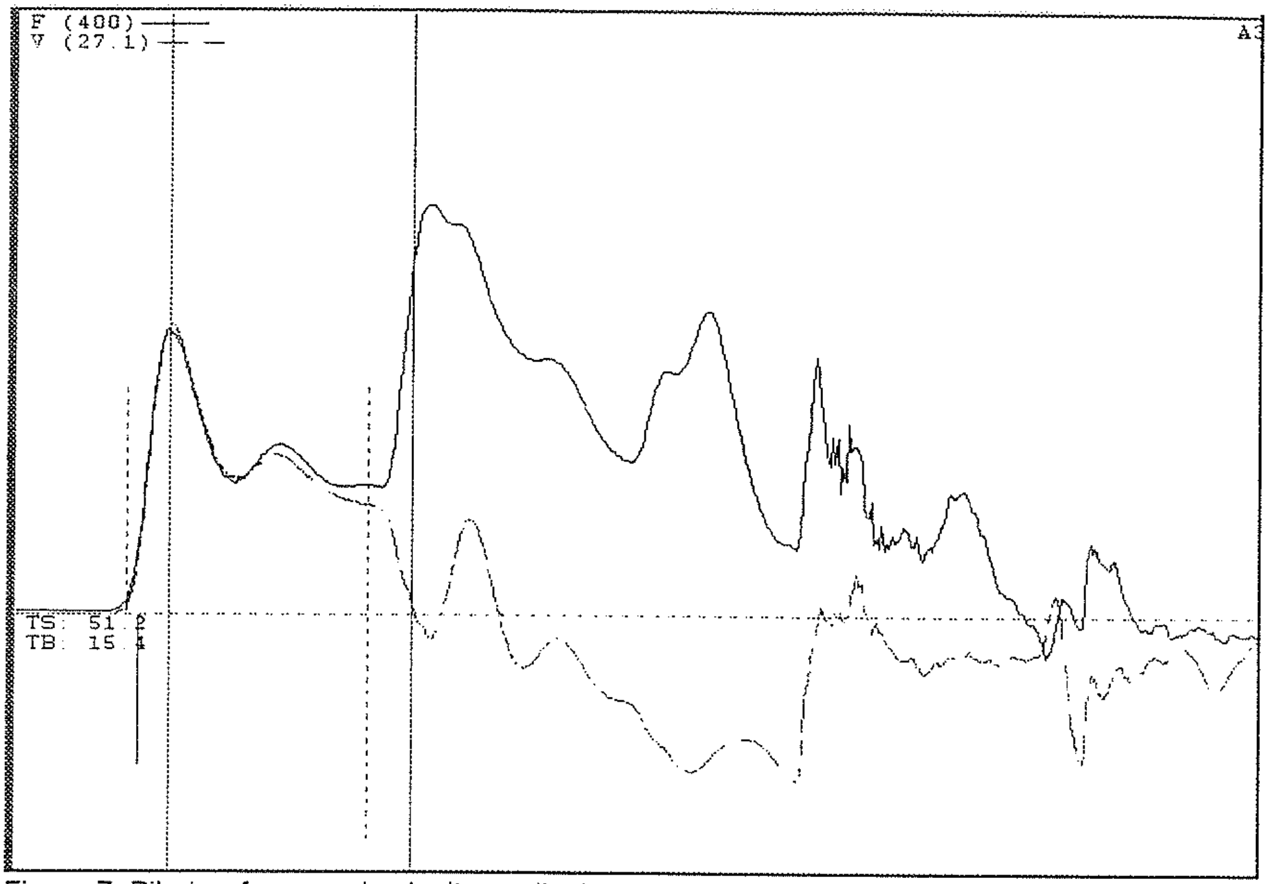


Figure 7: Pile top force and velocity as displayed on the PDA screen during installation

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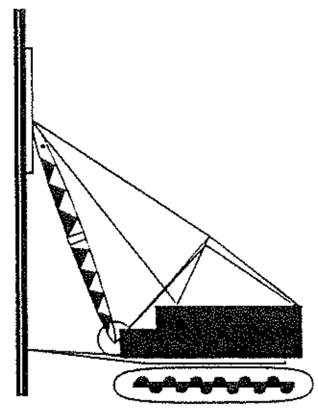

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