Modulus of Elasticity Impact on Equivalent Top-Loading Curves from Bi-Directional Static Load Tests

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ABSTRACT

The bi-directional static load test ("BDSLT") is widely used to test the geotechnical resistance of deep foundations. Many, if not all, of these tests are performed on instrumented drilled foundations where applied loads, strains, and displacements are measured during the load test. After the test is completed, the measured data are analyzed to determine required parameters for construction of the Equivalent Top-Loading ("ETL") curve. One of the main parameters used for the data-reduction process is the drilled foundation's composite-section elastic modulus, E. This parameter directly impacts the foundation's computed internal forces, ETL curve, and elastic compression. Several published methods have presented graphical and theoretical expressions to aid determination of a foundation's elastic modulus, whether a composite-section modulus, or the modulus of concrete/grout. This paper presents a review of these methods, and explores the impact of each outcome on the BDSLT results. Two concrete unconfined compressive strengths were considered to analyze the results of a BDSLT on a 72-inch-diameter drilled shaft. The concrete elastic modulus was determined from a method prescribed by the *American Concrete Institute* ("ACI"), and the composite-section elastic modulus was determined using the Tangent Modulus ("TM") method. Parametric studies were performed using load-transfer (*t-z*) analyses to investigate the effect of different moduli of elasticity on the *t-z* curves and the constructed ETL. Results of analysis showed that a given displacement of 2 inches and a 12 to 16 percent difference in axial rigidity, resulted in a 550 to 1200 kips difference in predicted load determined using the ETL curve. Similarly, a given load of 6000 kips and same percentage change in the axial rigidity, resulted in 0.5 inch to 1.18 inches change in displacement.

Keywords: Bi-Directional Static Load Test, Equivalent Top-Loading Curve, Modulus of Elasticity, Axial Rigidity, Tangent Modulus, *t-z* and *Q*-z analysis, ACI Formula,

Introduction

Bi-directional static load testing ("BDSLT") has become ordinary to evaluate the geotechnical capacity of deep foundations, particularly bored and drilled foundations. The use of an embedded hydraulic jack assembly to apply loads upward and downward aids to better understand the foundation internal force distribution, as well as the shaft and base resistances activated during the testing. In an instrumented BDSLT, strain measurements are obtained using strain gages installed along the foundation element, and the measured values are used to calculate foundation internal forces. From the stress-strain principle and mechanics of materials, the internal force at the strain gage level is calculated using the foundation cross-sectional area, A, and the foundation composite-section elastic modulus, E.

The cross-section area of the foundation element is likely well-known when the load test is performed on a driven pile. However, in the case of drilled shafts ("DS") and augered cast-in-place piles (ACIP), determining the cross-section area is likely not simple or direct. At a given location within the foundation element (e.g., at a strain gage level), using the theoretical diameter may not accurately determine the actual cross-section area. Similarly, the foundation's elastic modulus is often estimated based on the foundation's composite material properties, or calculated with defined methods (i.e., ACI). This paper is primarily focused on the influence of the elastic modulus on the results of a BDSLT.

Bi-Directional Static Load Test

Bi-directional static load tests are performed using a single or multiple expendable jack assembly embedded within the foundation element. Each jack assembly consists of a single or multiple hydraulic jack(s) located between upper and lower bearing plates. As hydraulic pressure is applied, the jack assembly can expand in both directions, and loads are applied to the foundation in an upward and downward direction. Depending on the design and purpose of the BDSLT, the jack assembly may be located at the foundation base, or at some distance above the base where the foundation upper-portion shaft resistance is equal to the foundation lower-portion shaft resistance plus base resistance (i.e., equilibrium point), Figure 1. To further observe and analyze the behavior of a foundation element under loading conditions, the foundation is instrumented using strain gages, telltales, and displacement transducers, Figure 1.

Strain gages installed along the foundation measure strains associated with each applied load, and aid to further determine the foundation internal force distribution. Telltales and displacement transducers measure displacement at various locations along the foundation. Foundation head displacement can be measured using displacement transducers, but is more-commonly measured using digital levels.

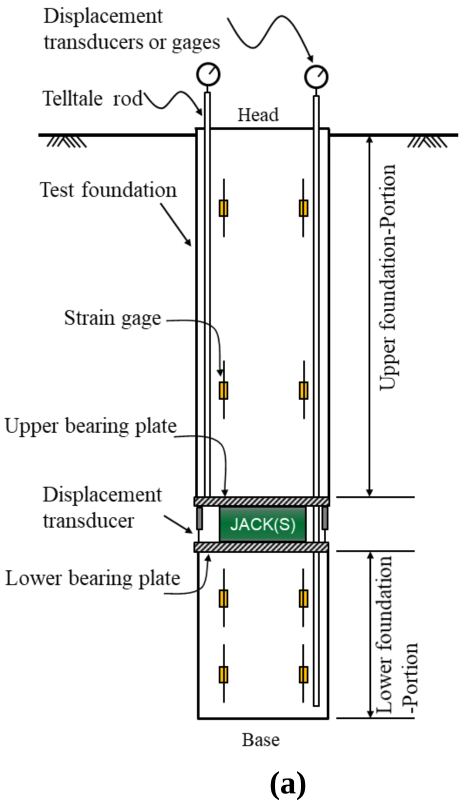

Figure 1. Bi-Directional Static Load Test arrangement **a)** Jack(s) at the equilibrium point and **b)** Jack(s) at the foundation base

BDSLT Procedure. After finalizing all drilling operations, the test foundation reinforcing cage with the jack assembly attached is inserted into the drilled hole, and concrete is placed. A sufficient time (as per project specifications or standards) has to elapse so that the concrete gains enough strength for testing. After the concrete strength is tested and approved for testing, pressures are applied incrementally to the embedded jack(s) to generate the bi-directional loading. During testing, the upper foundation portion provides reaction for test loads applied to the lower foundation portion, and vice versa (Brown et al. 2010).

It is important to note that one of the shortcomings of the BDSLT is the loading direction of the foundation upper portion. Test loads are applied to the bottom of the upper foundation portion, the opposite end as is loaded from service loads. Depending on stratigraphy, a BDSLT may result in stiffer soils at depth being loaded first, resulting in a stiffer foundation response than if top-loaded. Additionally, with top loading, internal loads at the jack assembly location are conveyed through the shaft, compressing it. Accordingly, top loading results in larger pile elastic compression in the upper foundation portion than a BDSLT. This limitation can be addressed by adding the foundation upper portion elastic compression to the measured displacements during equivalent top-loading (“ETL”) curve construction. With this exception of the elastic compression in the upper foundation portion, drilled shafts’ axial resistances exhibit no difference in behavior related to the loading direction (Brown et al. 2010).

Equivalent Top-Loading Curve

The ETL curve is an estimation of the foundation head load-displacement behavior which would result from a top-loading static compression test. Since bi-directional test loads are applied at some depth within the foundation, such load-displacement relationships are not measured but must be constructed. Depending on the load test results, three widely used methods can be considered to construct the ETL: The *Rigid Body* Method, the *Modified* Method, and the *Load-Transfer* Method, (Seo et al., 2016). The focus herein is on the load-transfer method.

BDSLT initial results are typically reported in a butterfly-shaped plot presenting load-displacement behavior of the jack assembly's upper and lower bearing plates, Figure 2. Upper bearing plate load-displacement behavior is governed by shaft resistance developed in the foundation upper portion; lower bearing plate load-displacement behavior is governed by shaft and base resistances developed in the foundation lower portion.

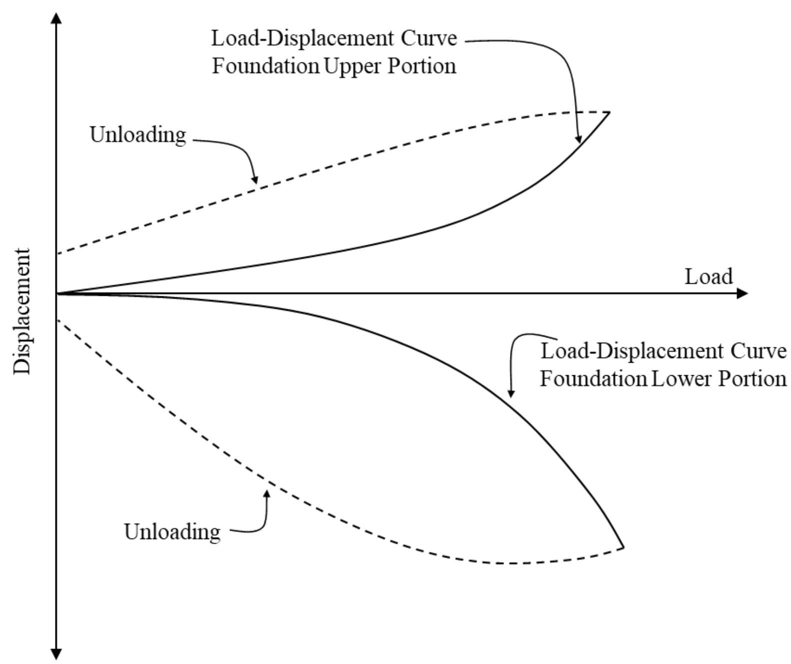

Figure 2. BDSLT initial results, (the "Butterfly" Curve)

The Load-Transfer Method (t-z and *Q-z Method).* The load-transfer method was developed by Coyle and Reese (1966) and further improved by Kwon et al. (2005). This method has the advantage that load and strain data are utilized to their fullest, and foundation elastic compression is considered by the analysis from the start. One disadvantage is that when ultimate resistance is not reached, extrapolations are required using hyperbolic curve fitting (England 2009). In the load-transfer method, the deep-foundation element is modeled as a series of segments supported by discrete nonlinear springs which represent the unit shaft or pile / soil interface resistance along the foundation (t-z curves), and total base resistance (Q_{BASE}-z_{BASE} curve).

Foundation internal forces can be calculated using the strain measurements at each strain gage level. From basic mechanics of material and Hooke's law, it is known that the relationship between force, F, at the strain gage level, and the material strain, ε, is defined as:

$$F = EA\varepsilon \qquad (1)$$

Where F is the foundation internal force, E is the foundation composite-section elastic modulus, A is the foundation cross-section area, and ε is the foundation segment axial strain. It is important to mention that within the deep foundation testing context, force is different than load. Load is external and applied to the top or bottom of a foundation or foundation portion and is measured; force is internal and is calculated based on foundation material properties as illustrated in Equation (1). Internal force is a result of external load. For each BDSLT load increment, foundation internal forces are calculated and plotted at discrete locations, Figure 3.

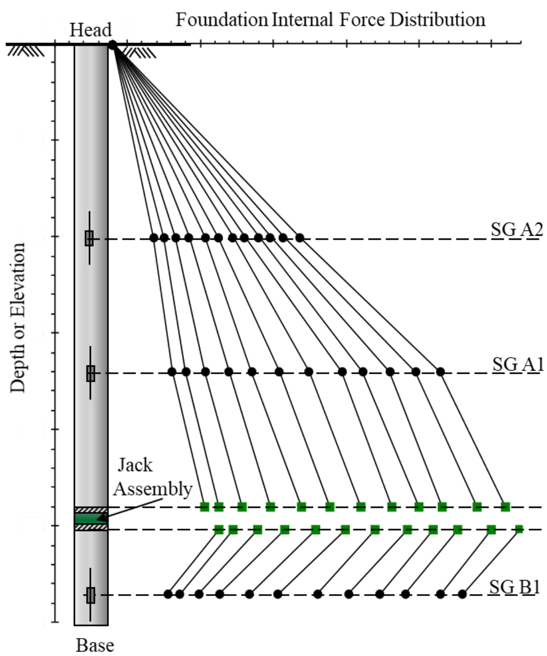

Figure 3. Foundation internal force distribution
SG: Strain Gage, **A:** above jack, **B:** below jack

From foundation internal forces, a series of *t-z* curves are obtained representing the shaft or pile / soil interface unit resistance behavior during loading. The difference in calculated internal force at the top and bottom of a foundation segment is divided by the foundation segment surface area to determine the average unit shaft or pile / soil interface resistance, t, which is plotted versus the segment's calculated midpoint displacement, z. From internal forces and displacements in the foundation lower portion, the total base resistance, Q_{BASE}, versus base displacement, z_{BASE}, curve is obtained.

To construct the ETL curve from the BDSLT results using the load-transfer method, base displacements under a top-loaded condition are first prescribed. Then, for each prescribed base displacement, the equivalent load at the foundation head which would result in the prescribed base displacement is calculated using an iterative process to achieve internal force equilibrium starting from the base element and moving upward to the last element at the foundation head. It is important to emphasize that although they exhibit similar behavior, the *t-z* curves determined from the BDSLT strain instrumentation differ from the relationship between relative shaft or pile / soil displacement and corresponding soil shear strength.

Elastic modulus

American Concrete Institute Method. The ACI 318-14 manual (2014) empirically estimates concrete elastic modulus based on the concrete's unconfined compressive strength and unit weight using the following expression:

$$E_c = 33\,\gamma_c^{1.5}\sqrt{f'_c} \tag{2}$$

Where γ_c is the concrete unit weight and f'_c is the concrete unconfined compressive strength, with Equation 2 results having units of pounds per square inch (psi).

For normal-weight concrete with unit weight between 90 and 160 pounds per cubic foot (pcf) (15 to 25 kN/m^3), Equation (2) can be written as:

$$E_c = 57{,}000\sqrt{f'_c} \tag{3}$$

The expressions presented by Equations 2 and 3 are the result of work presented by Pauw (1960) where the relationship between concrete unconfined compressive strength and unit weight was analyzed. During the era when this correlation was developed, average concrete strengths were significantly lower, potentially on the order of half the strength, of those encountered currently in practice. This concern has led scholars and industry experts to perform further research, and to develop new correlations (Ahmad and Shah 1985; Smith et al. 1964; Freedman 1971; Burg and Ost 1994; Iravani 1996; Mokhtarzadeh and French 2000 a & b). Although standard practice continues to follow the expression shown in Equations 2 and 3, synthesized descriptions of the above-mentioned research work, along with newer correlations, are published in ACI Committee Report 363 (2010). Furthermore, for foundations containing concrete or grout, the foundation elastic modulus is not constant during a static load test's loading cycle. Considering the large stress and strain levels applied to the foundation element during testing, the difference between the foundation's initial and final elastic modulus can be significantly different (Fellenius 2017).

Tangent Modulus Method. Some of the uncertainties associated with the ACI predictive method can be addressed by using the Tangent Modulus ("TM") method proposed by Fellenius (1989 and 2001), which determines the foundation composite-section elastic modulus from load test results. The TM method uses the stress-strain relationship to determine a strain-dependent expression for the composite-section elastic modulus at the strain gage level. The ratio of stress increment over strain increment, $\frac{\Delta\sigma}{\Delta\varepsilon}$ (i.e., the tangent modulus) is plotted versus strain for each strain gage level. After the shaft resistance between the applied test load and a given strain gage level is mobilized, this relationship becomes linear. The y-intercept of the best-fit line of the linear portion of the tangent modulus plot is the elastic modulus at zero strain, Figure 4. Particular (but not exclusive) to bored piles and drilled shafts, the linear expression for the strain-dependent composite-section elastic modulus may be unique to each strain gage level, Figure 4.

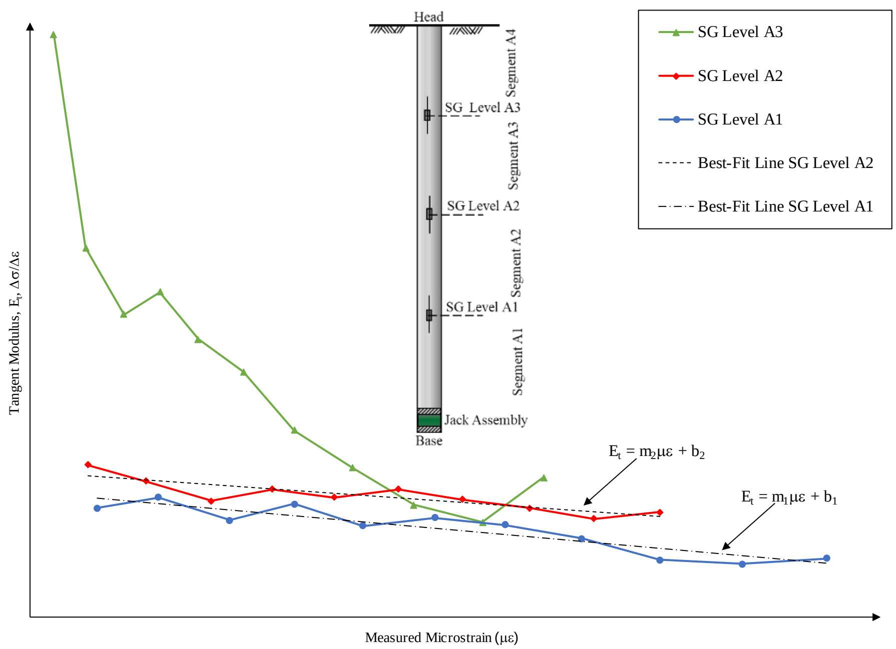

Figure 4. Schematics of the Tangent Modulus Plots and Best-Fit Lines

Analysis and Results

For the load test case selected for this manuscript, the elastic modulus was determined from the ACI method using Equation 3, and the TM method by creating the tangent modulus plots. For strain gage levels too distant from the applied load source which did not exhibit a linear portion of the TM plot, the linear strain-dependent expression for the composite-section elastic modulus corresponding to the nearer strain gage level that did exhibit a linear portion to the TM plot was used.

To illustrate the impact of the elastic modulus on the calculated *t-z* curves and the constructed ETL curves, results from a BDSLT were analyzed using varying values of E to generate the *t-z* and ETL curves.

The analyzed foundation consists of a 72-inch-diameter drilled shaft embedded 77 feet below ground surface. The predominant soil was classified as a silty sand (USCS Classification: SM) for the material surrounding the foundation shaft, and very dense sand at the foundation base. During load testing, strains were measured at three strain gage levels (SG Level A1 to A3) above the jack assembly, with SG Level A1 being closest to the assembly and SG Level A3 being the farthest from the assembly, similar to the foundation schematics shown in Figure 4.

The concrete mix design unconfined compressive strength was reported as 5,000 psi, with an average cylinder break strength reported as 7,360 psi for the load testing date (approximately 9 days after concrete placement), and 8,150 psi for 28-day break. It is recognized that measured strain values for 28-day concrete strength would have been different than for the as-tested 9-day concrete strength, and different *t-z* and ETL curves would have been developed. However, for purposes of illustration and instead of assuming an f'_c, the 28-day strength was selected to evaluate the impact of a higher concrete elastic modulus on the results.

The foundation axial stiffness, EA, is determined using the composite-section elastic modulus (E_{com}) and the foundation cross-sectional area, A. For the ACI approach, E_{com} is calculated using the concrete area elastic modulus obtained using the ACI method, Equation 3, and the steel area and elastic modulus, Table 1. Regarding the TM method, two of the strain gage levels did not exhibit any linear portion. In contrast, the SG Level A1 did exhibit a well-defined linear portion which was used for all strain gage levels. The E_{com} is determined directly from the applied load, strain measurements, and the best-fit line corresponding to the linear portion of the TM plot, Figure 5, Table1.

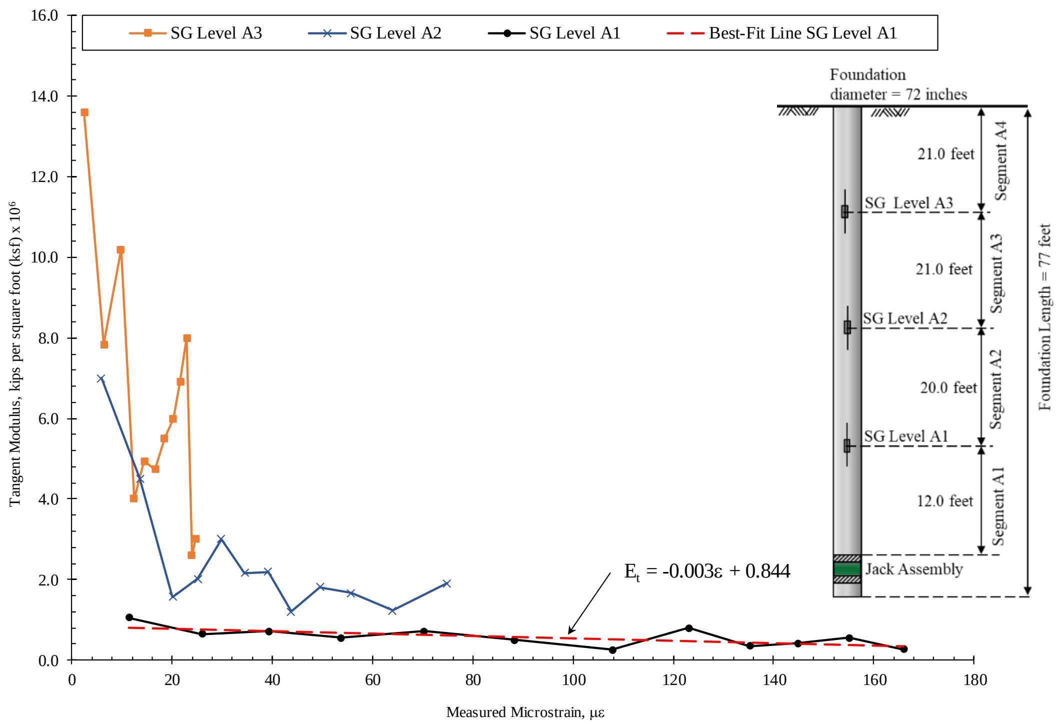

Figure 5. Tangent Modulus Plots and the Best-Fit line for SG Level 1

Table 1. Parameters used for the load-transfer method

Method	f'$_c$, psi (MPa)	E, psi (MPa)	E_{com}, psi (MPa)	EA, kipsx10^6 (MPax10^6)
ACI	6,360 (44)	5,046*(35)	5,290 (36)	21.5 (0.14)
	8,150 (56)	5,421*(37)	5,665 (39)	23.1 (0.16)
TM	6,360 (44)	NA	$E_t = -0.003\varepsilon + 0.844$	23.9** (0.17)

*Concrete elastic modulus; E_{com} is the composite-section elastic modulus; A is the cross-sectional area
**Calculated from the intercept of the E_{com} equation for the TM method ([0.844 x 10^6/144 in^2] x A)

Using the parameters shown in Table 1, the foundation internal forces were determined using Equation 1 and the unit shaft or pile/soil interface resistances, *t*, were calculated based on the segmental surface areas corresponding to the segments shown in Figure 5. The relationship between calculated *t* and its corresponding displacement, *z*, results in the *t-z* curves, Figure 6.

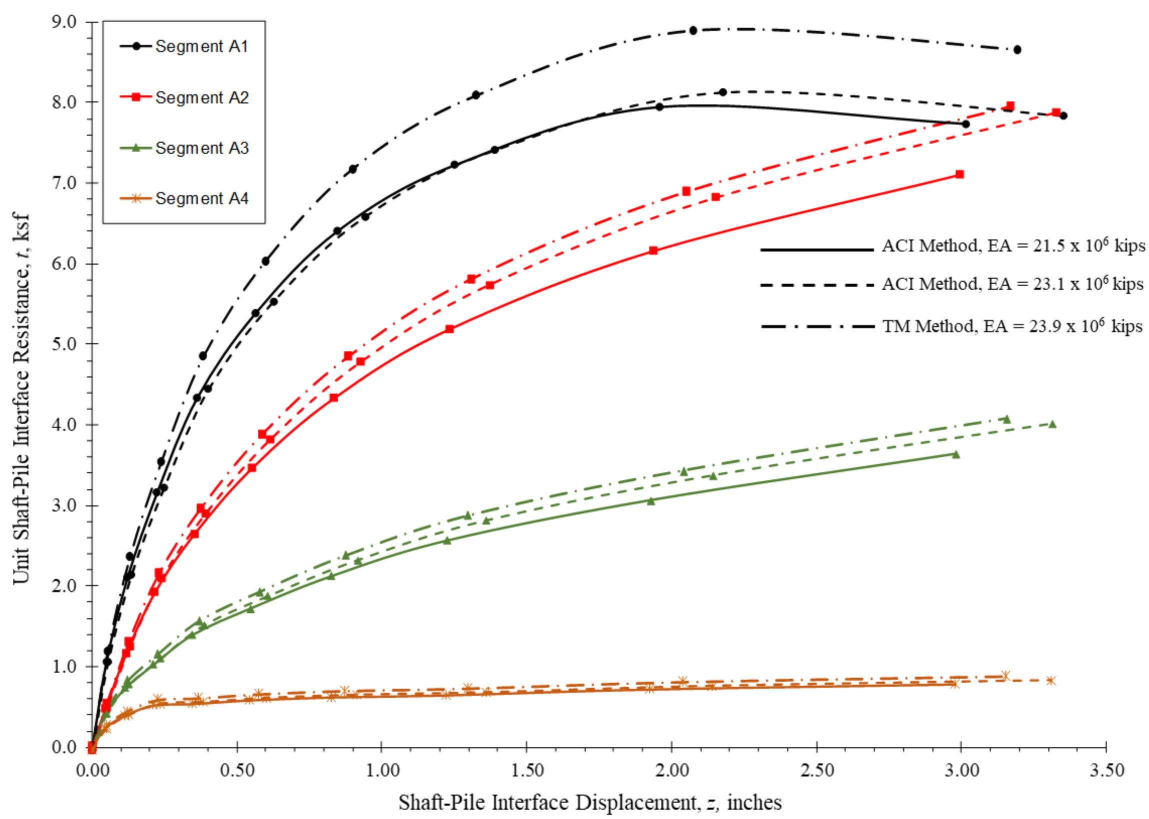

Figure 6. *t-z* curves generated based on three different values of composite-section elastic modulus, E_{com}

As observed in Figure 6, there are four *t-z* curves, each representing a segment along the foundation, plotted for the three different axial stiffness values, EA, shown in Table 1. When the foundation axial rigidity is increased, a given measured strain results in an increased calculated internal force at any given strain gage level, and a greater difference in calculated internal forces between adjacent strain gage levels. These greater differences in increased calculated internal forces between adjacent strain gage levels in turn result in increased values of *t*. Therefore, as the foundation's axial rigidity increases, calculated values of *t* also increase. In other words, the accuracy of the estimation of structural properties determines how closely the natural soil behavior can be predicted, but it doesn't govern natural soil behavior.

From the BDSLT test results, base resistances and corresponding displacements were plotted to obtain the Q_{BASE}-z_{BASE} curve. Using the load-transfer method and prescribed base displacements up to 10% of the drilled shaft nominal diameter, the ETL curve was constructed for all three stiffness values, Figure 7.

A review of Figure 7 indicates that although two of the ETL curves were constructed using very similar axial rigidities, the impact of the slight axial rigidity difference is significant enough to affect assessment of the load-displacement curve. In the case of the ACI-based axial rigidity, a given displacement of 2 inches and a 12 percent difference in axial rigidity (20.5x10^6 to 23.1x10^6 kips), resulted in a 550 kips difference in predicted load. In the case of the TM-based axial rigidity compared to the minimum of the ACI-based axial rigidity, a given displacement of 2 inches and a 16 percent difference in axial rigidity (20.5x10^6 to 23.9x10^6 kips), resulted in a 1200 kips difference in predicted load. Similarly, the impact of a slight change in the axial rigidity is significant enough to affect displacements. A given load of 6000 kips and 12 to 16 percent difference in the axial rigidity, resulted in a 0.20 to 0.5 inch displacement difference.

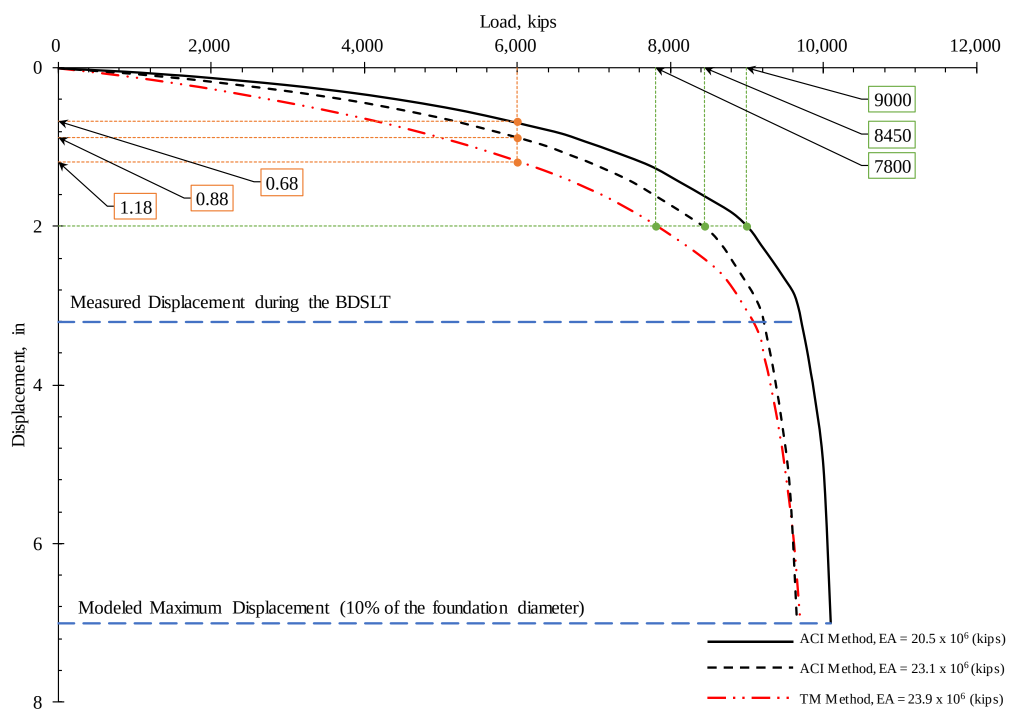

Figure 7. Constructed ETL curves based on different values of composite-section elastic modulus

Summary and Conclusions

This paper presents a review of two widely used methods to predict the foundation elastic modulus and explores the impact of the axial rigidity on the BDSLT results. Two concrete unconfined compressive strengths were considered to analyze the results of a BDSLT performed on a 72-inch-diameter drilled shaft. The concrete elastic modulus was determined from an ACI-based method, and the composite-section elastic modulus was determined using the TM method. Parametric studies were performed using load-transfer (*t-z*) analyses to investigate the effect of different moduli of elasticity on the *t-z* curves and the constructed ETL.

In the case presented in this paper, results of analysis showed that the impact of the slight axial rigidity difference is significant enough to affect assessment of the load-displacement curve. In comparing the two ACI-based axial rigidity, a given displacement of 2 inches and a 12 percent difference in axial rigidity, resulted in a 550 kips difference in predicted loads. In the case of the TM-based axial rigidity compared to the ACI-based axial rigidity, a given displacement of 2 inches and a 16 percent difference in axial rigidity, resulted in a 1200 kips difference in predicted load. Similarly, the impact of a slight change in the axial rigidity is significant enough to affect displacements. From the ETL curves a given load of 6000 kips and a 12 to 16 percent change in the axial rigidity, resulted in displacements of 0.68 inch and 1.18 inches.

Major objectives of bi-directional static load testing routinely include measuring strain at discrete levels within the foundation, from which computed internal force profiles, and *t-z*, Q_{BASE}-z_{BASE}, and ETL curves are produced. To determine these profiles and curves, the foundation axial rigidity, EA, at each strain gage level must be established. Axial rigidity can be determined by estimating the foundation's composite materials' individual elastic moduli and cross-sectional areas, or by estimating the product EA. A commonly used method to estimate concrete or grout elastic modulus is the ACI formula, which is empirical-based. The TM method provides a means by which to estimate the product EA (axial rigidity) at individual strain gage levels, and where applicable, is considered to provide more-accurate results than using the ACI formula. Differences in estimated concrete modulus or axial rigidity values result in demonstrable effects to the computed internal force profiles, and *t-z*, Q_{BASE}-z_{BASE}, and ETL curves.

Results from analysis has revealed that selection/determination of individual composite materials' moduli and/or composite-section axial rigidity, EA, can have significant enough impact on calculated major objectives of the bi-directional static load tests. Results affected by the slight change in the axial rigidity could misguide when technical conclusions are to be made from ETL curves. When applicable, methods which determine the product EA provide an improved analysis compared to moduli determined based on empirical relationships. Accordingly, a test engineer or designer should be aware of the effect and relative accuracy of these various analysis options, and employ appropriate data reduction methods accordingly.

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