A Rational and Usable Wave Equation Soil Model
Based on Field Test Correlation

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Abstract

Dynamic soil modeling of pile driving is presented. To improve the commonly used model, both static and dynamic data have been measured with the Standard Penetration Test (SPT) and a data base with correlations to full scale pile tests was generated. The literature was investigated for so-called rational soil models whose parameters can be derived from standard geotechnical soil properties and a correlation study was made to relate standard soil constants with dynamic model parameters.

Several ideas for model improvements were found in the literature. However, these suggestions were complex and results not proven by measurements. A simple, improved dynamic soil model for pile driving which has been compared with dynamic testing of both the full scale pile data base and SPT results and how it can be implemented into standard wave equation practice are discussed.

Introduction

The main pile driving question is how to quickly, safely and economically drive a pile to sufficient capacity with acceptable settlements. This paper attempts to explain the basics of past modeling efforts and why changes should be made to existing technology. Smith (1960) devised the current pile driving analysis model which is successfully

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used in many countries and commonly referred to as the "wave equation". His goal was to replace pile driving formula (based on energy concepts) relating bearing capacity to blow count with a more accurate numerical algorithm. Wave equation modeling uses a one dimensional mathematical representation of hammer, driving system, pile and soil which allows an accurate calculation of (a) the progress of pile penetration into the ground, (b) the relationship between pile bearing capacity and pile penetration, (c) the stresses in the pile during driving and (d) the mechanics and/or thermodynamics in a hammer. Smith did not consider the driveability problem which includes blow count and stresses as a function of pile penetration.

Many subsequent efforts at improvement were directed at a soil model which (a) can be physically explained and (b) whose component parameters can be derived from standard geotechnical engineering soil properties. Several theoretical studies are primarily based on Novak et al. (1978) who derived soil stiffness and soil damping from the soil shear modulus. Randolph and Simons (1986), Chow, et al. (1988) and Lee, et al. (1988), included this concept in their proposed soil resistance models. Related models were described by Corte and Lepert (1986), Holeyman (1985), Middendorp et al. (1984) and others.

Smith's model relates ***elevated resistances*** due to high loading rates with a velocity dependent resistance in addition to the displacement dependent, static resistance. Coyle and Gibson (1970) used the same concept, but with a dynamic resistance varying exponentially with velocity as determined by laboratory testing. Briaud and Garland (1984) used a time to failure or an average loading velocity raised to some power to define a ratio of total dynamic capacity to static capacity. The maximum applied load is also displacement dependent. None of these models proposed by the academia have been subjected to extensive correlation with a database of field results and thus remain unproven. However, there is unanimous agreement that the current practice may lead to errors particularly for situations which are beyond the traditional data bases established with hammers of relatively low impact velocities.

Finally, significant errors in dynamic pile predictions are made because of an inaccurate assessment of the losses or gains of soil strength caused by pile driving (*e.g.*, Heerema, 1979, Svinkin et al. 1994). Skov and Denver (1988) proposed the direct measurement of these effects; however, the prediction of soil strength changes with time is still very difficult. A proposal on implementing these effects in driveability analyses will be made.

Background of Problem Statement

Several phenomena contribute to the behavior of the soil during pile driving and each must be clearly understood and accurately modeled if the analysis is to properly predict the pile driving process. The major effects are velocity or rate dependent effects, soil movement, soil degradation or set-up, and creep. These effects will be discussed individually.

Rate effects

The ***static resistance*** is a function of the relative pile-soil displacement. Unfortunately, the soil resistance does not behave identically during static and dynamic load applications. When loads are applied rapidly as in pile driving, additional velocity and acceleration dependent resistance components are generated. These ***dynamic resistance*** components increase the apparent resistance of a quickly penetrating pile compared to a slowly advancing one. In this paper, the ***total resistance*** or ***elevated resistance*** is the sum of static and dynamic resistance.

For long piles with resistance distributed along the shaft and for any pile with high resistance, the maximum velocities along the pile shaft may be highly variable and generally much lower than at the pile top. Under these conditions, the non-linearity of dynamic resistance *vs.* velocity becomes very important and would require very high damping factors with traditional linear damping models. Conversely, new model hammers with higher strokes or greater efficiencies produce much higher velocities than contained in the original data base used to develop parameters for the original wave equation model. An improved method of accounting for rate effects appears to be desirable.

Soil movement

Smith made a simplifying assumption that the soil is fixed in space. Soil motions can be included in the calculations with a so-called radiation damping model (the soil motion radiates energy away from the pile soil interface). Unfortunately, ***radiation damping*** and ***viscous damping*** are terms which are often used interchangeably. CAPWAP (Rausche, et al. 1985, GRL 1993 and Rausche et al. 1994).contains a radiation damping model and extensive parameter studies indicate certain narrowly bounded model parameters for good correlation with static load test capacities. However, attempts to incorporate this new model into the wave equation analysis GRLWEAP (GRL 1991) have not succeeded because of sensitivity of the calculated blow count to the radiation damping parameters. Therefore, the soil in the model proposed in this paper is considered fixed.

This model has limited accuracy for refusal cases such as driving into hard rock or dynamic loading of drilled shafts, ***etc.*** where soil motions may be as large as pile motions.

Static soil resistance degradation (set-up)

When a hammer strikes a pile, soil particles around the pile are suddenly displaced as the pile penetrates under a hammer blow. Moreover, the pile also rebounds a considerable distance. In fact, during hard driving, the upward and downward pile movements are much greater than the net permanent penetration into the ground due to the elasticity of pile and soil. This relentless down and up pile motion affects the ground pore water pressures and/or destroys the natural fabric of the soil, at least temporarily. The resulting loss of soil strength leads to a static soil resistance which is less than the long term value under sustained loads. The soil resistance generally increases after driving and therefore the term *set-up* describes what is really only a return to a long-term strength as might be calculated from a static analysis. Usually, a set-up factor is used to calculate the long term capacity from the temporarily reduced capacity at the end of pile installation using an exponential approach such as proposed by Skov and Denver (1988).

$$R_u = R_o[1 + A \log_{10}(t/t_o)] \qquad (1)$$

In this expression, A is a non-dimensional quantity defining the capacity increase between time t_o (when the capacity is R_o) and time t.

Creep

Compared to short term static or dynamic loads, a pile will experience greater settlements due either to soil creep or to soil consolidation under loads maintained for a considerable time. Under short duration dynamic loads and quick static tests, these so-called secondary settlements are hardly noticeable. However, under long term loads they may affect a pile's serviceability. Moreover, in evaluating a static load test, creep deformations may make the apparent capacity lower for a maintained load test than for a quick test. Therefore, for a correct prediction of a pile's load-set behavior, an estimate of the creep deformation as a function of loading rate should be added to the dynamically predicted values perhaps using an exponential expression. Unfortunately, little has been done to solve this problem.

Description of the Improved Model

Basically an extended Smith model is proposed as in Figures 1a and 1b for pile shaft and toe soil resistances respectively. The models represent the forces in the pile-soil failure zone. Thus, radiation damping is ignored. The components of the models are described below.

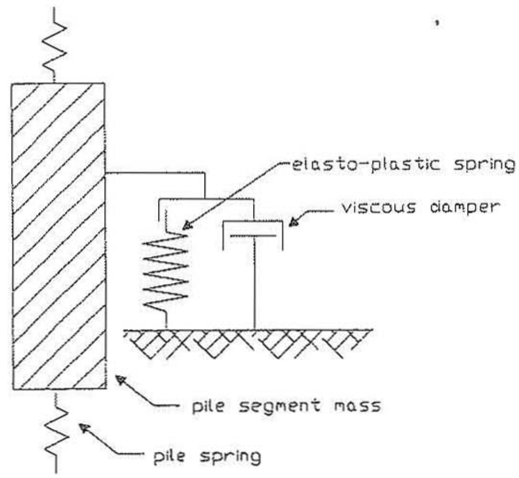

Figure 1a. Shaft Resistance Model

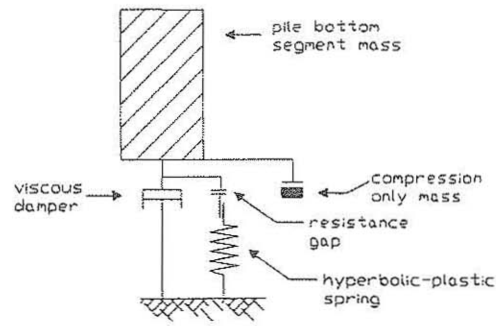

Figure 1b. Toe Resistance Model

Consideration of rate effects and damping

Compared to a slow (static) penetration into the ground, the dynamic resistance may be higher than the (possibly temporarily reduced) static resistance due to the inertia of the displaced soil particles, and due to the high relative pile-soil velocities. Smith considered these velocity effects using a simple approach based on the pile velocity, v.

$$R_d = J_s R_s v \quad (2)$$

where R_d is the dynamic resistance, J_s is the Smith damping factor and R_s is the static pile soil resistance which is a function of time. The total resistance, R_t, that the pile has to overcome is then

$$R_t = R_s(1 + J_s v). \quad (3)$$

Many correlation studies, summarized by GRL (1992), have shown considerable scatter for the damping parameter, J_s, using Smith's approach. No direct relationship between J_s and soil type has been clearly observed. In fact, the damping term is often adjusted to absorb all of the uncertainties in a correlation study. For example, where inertia effects, soil set-up and relaxation (including different times of pile installation, static load test and/or restrike test) and incomplete capacity activation (high blow counts) have not been properly considered, improper corrections are often made to the shaft damping parameter rather than to the computed capacity value.

Coyle and Gibson (1970) suggested that the maximum dynamic resistance contribution varies not linearly but rather exponentially to the pile velocity. Thus,

$$R_t = R_s(1 + J_G v^N) \tag{4}$$

where the exponent, N, typically is less than 1. While this seems inherently good in that it matches laboratory measurements of **maximum** damping resistance, actual application of this equation for the wave equation creates large unrealistic damping oscillations when the velocity during the unloading portion of the blow. Note also that the damping constant J_G in this approach has dimension $[s/m]^{1/N}$. A conversion of a damping factor from Smith's linear system to a nonlinear system is therefore not a simple operation.

A somewhat different approach (Briaud and Garland 1984) relates the total dynamic capacity to the static one using a "static" velocity, v_s (*e.g.*, the load test velocity).

$$R_t = R_s J_c (v/v_s)^N \tag{5}$$

There is a very basic difference between Briaud's approach and those proposed by Smith and Coyle. Both Smith and Coyle approaches include a separate damping component, usually explained as related to the viscous behavior of soil; for Smith and Coyle, the elevated resistance is simply the sum of the static and the damping components. Briaud's approach determines an overall elevated resistance, instead of defining individual static and damping components. If Briaud's elevated resistance is determined by a dynamic test, then it must be reduced to a long term static value using the ratio of testing velocities which are themselves time variable. To be useable, Briaud's method requires an average velocity, *i.e.*, the failure set divided by the time expended to reach failure. An additional damping component must still be added as per Randolph to produce overdampened behavior seen in dynamic pile test records.

In order to satisfy the need for (a) an elevated resistance, (b) viscous damping and (c) an exponential relationship between loading velocity and capacity Rausche et al. (1992) proposed the following equation.

$$R_t = R_s \left[1 + J_R \, v_x^N \, \frac{v}{v_x} \, \frac{R_a}{R_s} \right] \qquad (6)$$

This proposed approach is the improved model and contains only one variable in the damping term, namely the pile velocity, v. The second term in Eq. 6 represents linearly viscous damping. The resistance R_a is the maximum static resistance component activated during the blow prior to the time under consideration (starts at zero for every blow, increases until the failure load is reached, and then remains constant). Similarly, v_x is the maximum velocity achieved up to a particular time during the blow. Both R_a and v_x usually reach their maxima during the blow very quickly and then remain constant. This approach addresses completely an elevated resistance and the exponential nature of the maximum viscous damping as determined in laboratory tests, but avoids the numerical shortcomings of the Coyle approach since the exponential term, v_x, does not return to near zero during the blow.

In any event, the damping factor, J_R, and the exponent, N, must be determined from special laboratory or in-situ tests or from values given in the literature. J_R has dimensions $(s/m)^{1/N}$ and determines the magnitude of the viscous damping force. The velocity exponent, N, defines the rate at which the damping increases, given a velocity maximum, v_x, which is related to the measured velocity, v.

This improved exponential model yields results comparable to Smith's approach when the maximum pile velocity is within certain narrow ranges. An approximate recalculation of the required damping factor J_R for the exponential approach from the corresponding Smith damping factor J_S commonly used is easily and automatically possible. For example, after choosing a reference velocity v_x (say 3 m/s) and a ratio of average temporary to activated static resistance (say 0.9), J_R could be calculated from

$$J_R = J_S \, (0.9)_3^{1-N} \qquad (7)$$

Details of Resistance-Displacement Relationships

Shaft

The elasto-plastic Smith's shaft resistance model is satisfactory. Although it has been proposed by Novak and other researchers that the quake (the elastic dynamic relative pile-soil displacement) be determined from the shear modulus of the soil, numerous dynamic signal matching analyses have shown that Smith's proposed shaft quake of 2.5 mm (0.1 inches) is generally reasonable. Thus, Smith's original shaft quake appears to be better than one that relies on the soil's shear modulus which is strongly dependent on the magnitude of the shear deformations.

The authors also investigated a modified, bi-linear shaft unloading quake to introduce extra hysteresis into the static shaft resistance law. However, the complexity of the numerical treatment and an additional unknown did not justify the small gain in realism.

Toe

Correlation studies found in the literature as well as signal matching by CAPWAP have not shown a relationship between the soil stiffness (flexibility or quake) and soil type. In fact, the only conclusions supported to date are that high dynamic (not necessarily static) quakes occur sometimes in saturated soils (Likins 1983) and quakes larger than the GRLWEAP recommended D/120 are often observed. A hyperbolic toe resistance vs. toe displacement relationship is perhaps more realistic and can be numerically achieved by introducing a factor, c_q, which multiplied with the quake yields the point where ultimate is reached (Figure 2). Thus, unlike a pure hyperbola, the improved model has an ultimate resistance that still can be reached at a finite toe displacement. This load-movement relationship had been proposed by Eriksson (1990).

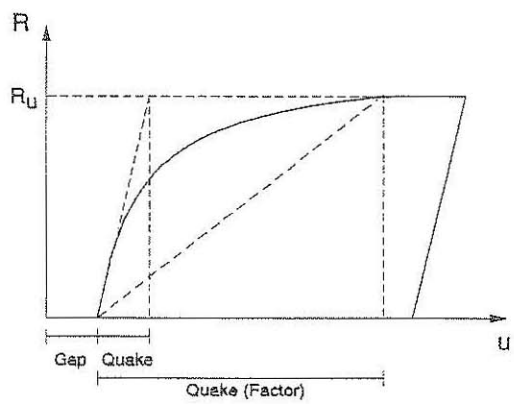

Figure 2. Toe Static Resistance Penetration Behavior

Toe Soil Mass Resistance

Force and motion measurements at the pile toe have indicated a pronounced inertia effect in cohesive soils (Grasshoff 1953 and Rausche 1970). Figure 3 shows a similar effect observed in a modified SPT; dynamic strain and acceleration records were measured at the SPT top and the toe resistance force and toe displacement were calculated. The observed inertia effect is the result of a soil mass adhering to or moving with the SPT special tip; the GRLWEAP toe mass model is only active during the first positive acceleration. Using this model, the viscous damping was reduced in a number of modified SPT tests and their subsequent data analysis. These special SPT tests were performed to confirm suspected soil behavior and are probably not necessary for standard SPT measurements.

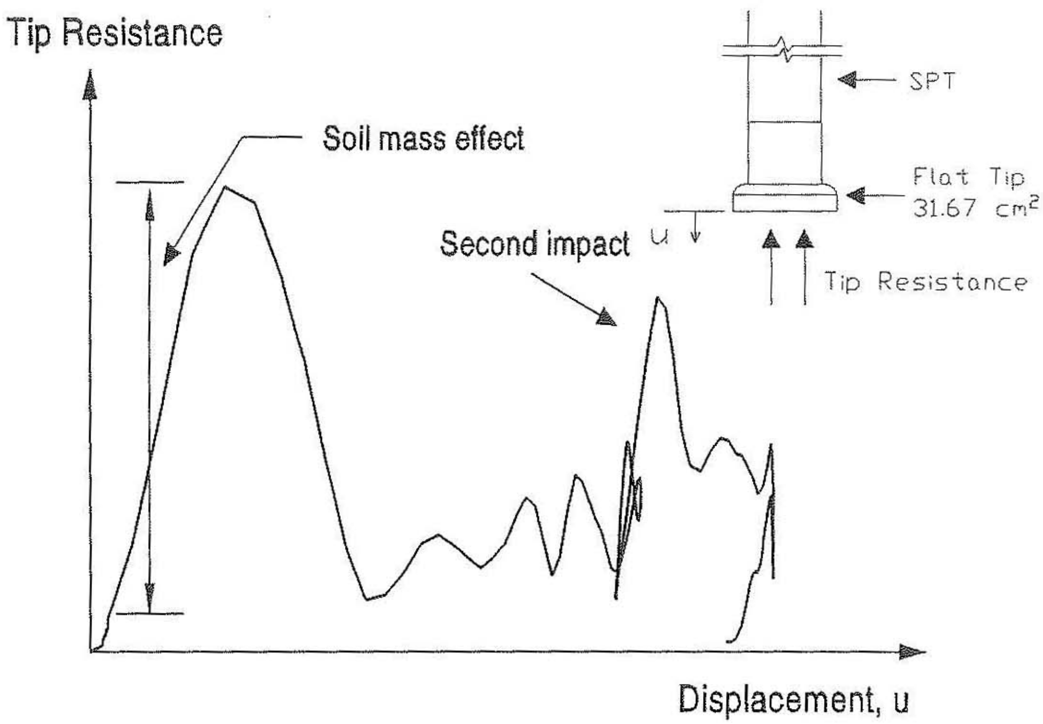

Figure 3. Resistance Force *vs.* Displacement at Special SPT Tip from Dynamic Measurements

Consideration of Soil Strength Changes During Driving

A hammer blow changes the soil strength due to pore water pressure and other effects mentioned earlier. Regaining the original soil strength occurs in an logarithmic manner. Skov and Denver (1988) suggested two restrike tests, one early and one later, to allow for an extrapolation to the pile's long term bearing capacity; they also recommend the first restrike be no earlier than 12 hours after driving for a reliable prediction of long term capacity. Their formula (Eq. 1) therefore cannot be reversed to precisely predict the capacity during driving, but does indicate the strength change trend during driving that can be modeled.

Static resistance at the time of driving from presumably known long term resistance should take into account the energy expended on the soil. Such an approach would be beneficial both for impact and vibratory driven piles. It also must take into account the "rest periods" between energy dissipation in the soil (Figure 4). Thus, if at one point along the pile the ultimate shear resistance is τ_u, then the unit energy dissipated in the soil by pushing the pile relative to the soil a displacement u is

$$e_{soil} = \tau_u(u) \tag{8}$$

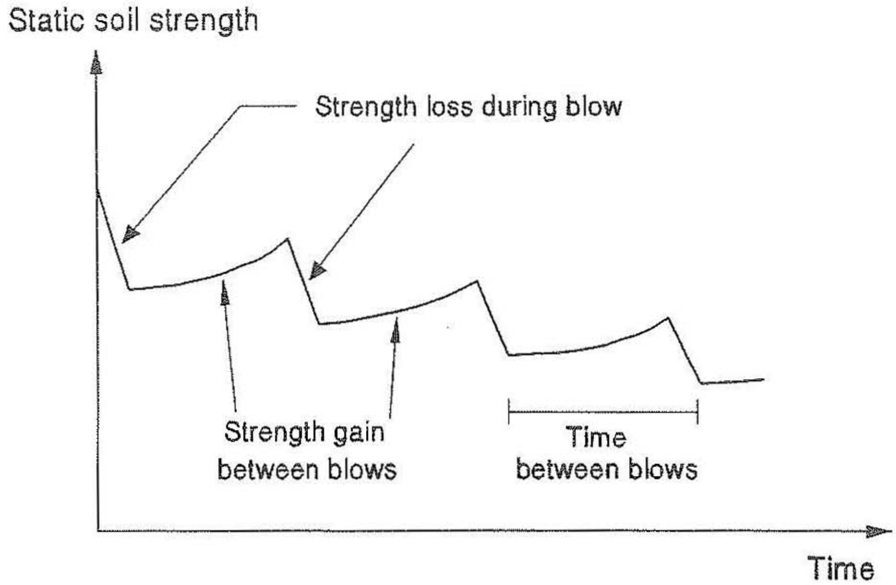

Figure 4. Soil Strength Changes During Driving

There is a certain limit energy, e_l, which once reached or exceeded, causes the soil to reach its residual ultimate strength τ_{ru}. Conceptually, each time energy is dissipated in the soil, the temporary ultimate shear strength of the soil reduces by an increment which, as a first approximation can be considered linearly related to the current shear strength (although some exponential law may also be appropriate).

Thus,

$$\Delta\tau = \frac{e_{soil}}{e_l} \tau_{old,u} \tag{9}$$

$$\tau_{new,u} = \tau_{old,u} - \Delta\tau \tag{10}$$

with

$$\tau_{ru} \leq \tau_{new,u} \leq \tau_u \tag{11}$$

During any pause in energy application, such as between hammer blows or interruption of driving, the soil regains strength. As a time period of rest, Δt, passes, the shear strength increases to

$$\tau_{new,u} = \tau_{old,u}[1 + A \log_{10}\{(\Delta t + t_{lag})/t_{lag}\}] \quad (12)$$

where t_{lag} is an appropriately chosen lag time from the beginning of the most recent hammer blow and A is as discussed for Eq.1. If successful, this approach would make driveability analyses much more accurate.

Static soil resistance increase (relaxation)

The static (displacement dependent) soil resistance component may increase temporarily during dynamic loading. A good example is the occurrence of negative pore water pressures at the pile toe in very dense saturated fine sands and silts. After driving, pore water pressures and effective stresses return to their natural levels and long term resistance then is lower; the term ***relaxation*** has been used. Thus, by the time a static test is performed on a dynamically installed pile, relaxation effects usually have occurred and the engineer is left to wonder why the driving resistance was so high. There is no current mathematical model describing this relaxation effect, since it happens very quickly. However, an approach similar to the proposed approach for the soil resistance degradation should be applicable since pore pressure dissipation is generally logarithmic. Relaxation has also been observed for piles driven into weathered shale, although there the time required is longer and the logarithmic equation seems applicable.

Recommended Parameters and Procedures

For the shaft, if no experience data exists to suggest otherwise, the toe quake should be fixed at 0.1 inches. As long as no other experience data exists, the proposed exponential shaft damping approach (Eq. 6) should be used as long as no other experience data exists. The shaft damping factor J_R can be computed from Eq. 7 based on the Smith parameter J_S chosen from soil type according to Smith's original recommendations.

The toe quake recommendation of D/120, independent of soil type, still seems to be reasonable. It is recognized, however, that this value is usually a lower bound. In fact, quakes on small diameter toes like the modified SPT of Figure 3 suggest substantially greater values than D/120. More realism is therefore introduced with the hyperbolic loading behavior. The displacement at which the hyperbola ends and where pure plasticity starts should be 2.5 times the toe quake unless more specific data is available for a site. The soil mass may be calculated for open profiles considering a volume that has a diameter equivalent to that of the pile and a height of 2 m unless there is reason to believe that the soil column is actually shorter. For large displacement piles a soil mass attached to the pile bottom (practically extending the pile) may also be calculated based on a volume that equals a cube with its dimensions equal to the effective pile diameter. Toe damping would be replaced in part by the inertia of the soil mass for cohesive soils. The velocity damping force therefore can be smaller and toe damping parameters can be fixed, independent of soil type and would use the exponential approach as detailed above for the shaft damping. This approach is in general agreement with results from signal matching analyses of numerous field tests.

Distinction has to be made between the bearing graph analysis and a driveability study. Bearing graphs still may be calculated based on assumed capacity values split into shaft resistance and end bearing components. For driveability analysis, the data preparation process requires the ultimate unit static soil resistance, perhaps obtained by measurement on an SPT for improved predictions. While the Smith approach was based on resultant force values, more realism and accuracy can be expected (particularly for non-uniform piles) when the capacity calculations are based on unit shear stresses and an equivalent circumference for both shaft and toe. With this additional input information, the calculation of resultant resistance forces and of a soil mass size may be automated.

For the driveability analyses, the soil degradation/set-up/relaxation effects can now be automatically considered. If proven adequate, this concept would revolutionize the currently available analysis process.

Summary

A modification of Smith's soil resistance model has been proposed. For the practicing engineer, these changes will not require any additional knowledge of soil behavior than currently required, although for driveability analyses, some measurement on a SPT will be useful. Naturally, the more accurate the soil exploration, the more accurate the prediction of the static and dynamic soil behavior.

The proposed model considers the exponential nature of the total soil resistance increase with loading rate. It also considers a static and a dynamic resistance component rather than one increased displacement dependent total soil resistance. Furthermore, the model does not include radiation damping as an additional refinement. The model does consider the hyperbolic nature of the resistance *vs.* toe penetration behavior, and toe soil mass effect. This model does <u>not</u> aid in the calculation of plug-pile interaction forces.

This improved model for the analysis of pile driving has been selected such that the engineer is not burdened with a totally new approach or complex additional calculations for input preparation. The additional model parameters, exponent, N, toe quake factor, c_q, and soil mass, m_t, can be easily estimated or, for many standard analyses, ignored. The new damping factors can be calculated based on current practice. An effective circumference, providing the pile-soil contact area for both shaft and toe, is a known quantity. With unit shaft resistance and end bearing pressures (perhaps measured) specified driveability analysis become a simple and realistic task.

Recommendations

The model presented here must still be extensively tested. Although it requires some additional input (exponent, toe quake factor, toe soil mass, effective pile circumference), comparison analysis would be very quick and easy. At the same time, however, existing measurement capabilities and the information available during acquisition of SPT data should be used.

After the completion of the dynamic analysis, a rational and realistic static reanalysis should be performed in the future. This analysis would yield the pile top load-set curve and therefore would allow for an easy check of the accuracy of the dynamic simulation. For the greatest accuracy, estimated effects of soil set-up/relaxation (and possibly even creep) should be included and checked against real measurements.

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