Wave Equation Soil Constants From Dynamic Measurements on SPT

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Abstract

The widespread use of Standard Penetration Tests (SPT) for subsurface investigation makes it economically desirable to extract from the test more quantitative soil strength information than merely an N-value. One technically feasible method proposed in this paper would require the measurement of force and motion at the drill string top during SPT sampler driving. As with dynamic pile testing methods, these measurements can be evaluated for the transferred energy to the drill rod, and the static and dynamic soil resistance components. For SPT, the procedure to obtain resistance is even simpler than for piles since the test is performed in an open drill hole, and the only driving resistance acts at the sampler. Therefore, it is possible to calculate in closed form the force and motion at the sampler from the top measurements. Together with the knowledge of soil type and SPT N-value, the static and dynamic resistance components and the shaft and/or toe wave equation soil constants of damping factor and quake can be calculated.

Motion during the SPT impact event is measured by accelerometers. In the past, difficulties were sometimes experienced with acceleration measurements. Modern instrumentation technology makes these difficult measurements very reliable, allowing for routine dynamic testing during SPT sampler driving. However, it is advisable to automatically check the records by double integrating the acceleration record, and, if necessary, adjust the acceleration measurements. The method also calculates maximum energy transferred to the drill rod, finds the sampler force and motion, and then derives the wave equation soil constants. Example SPT Test calculation results obtained from cohesive and cohesionless soils are also presented.

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Introduction

The Standard Penetration Test (SPT) is the most widely used subsurface investigation tool in the United States and in many other countries. The test has the advantages of producing a soil sample, in addition to quantitative soil strength information, the N-value. While the test has been standardized by ASTM D-1586, it is possible to perform the test with a variety of equipment and questions have been raised as to the reliability of the N-value given variable energy transfer to the drill rods depending on the SPT equipment. Schmertmann (1979) has suggested that the SPT N-value is inversely proportional to the energy transmitted to the drill rod; and therefore, the N-value could be adjusted to any desired standard energy if the true energy transfer is known. Thus ASTM D-4633 was written standardizing the measurement of force in the rod and the calculation of transferred energy from force under certain simplifying assumptions.

Strictly speaking, energy determination in the rod requires the measurement of both force, F, and velocity, $\dot{u}$, at the top of the drill rod ($E = \int F\dot{u}\, dt$). The force is best measured with resistance strain gages glued to the rod, while velocity is determined from integrating signals from accelerometers rigidly attached to the rod.

However, for a truly uniform rod with no shaft resistance, the force equals the velocity times the impedance (EA/c, where E is the material modulus, A the cross section area, and c the material wave speed) and a simplification can be made to a computation using only a single measurement ($E = [c/EA] \int F^2\, dt$). However, there are many situations under which the calculation of energy from force alone is erroneous. Since in the past, difficulties had been experienced with the acceleration measurements, the basic shortcomings of computing the energy from the square of the force were accepted. Measurement difficulties were caused by metal-to-metal impacts which generated high frequency signals whose g-levels often exceeded the accelerometers' specifications (Hauge, 1979). Today, further improvements in accelerometer technology make possible a reliable measurement of both force and acceleration and allows for routine dynamic testing during SPT sampler driving. Thus, energy should now be measured with the preferred, theoretically correct, force and velocity method ($E = \int F\dot{u}\, dt$).

For a uniform rod, top force and velocity (integrated from acceleration) allow for the calculation of the force and velocity at the sampler since there is no shaft resistance along the drill rod. Teferra (1977) showed that the dynamic force *vs* displacement curves at the pile toe are similar to static ones. Thus, the sampler forces and motions can be used in evaluating the static and dynamic resistance components of the soil at the sampler. These values can then be converted to soil resistance parameters for wave equation analyses of full scale piles. Rausche et al. (1996) have shown correlations between SPT predicted pile behavior and both observed blow counts from pile installation and statically measured load-set curves.

The method presented in this paper has been programmed for automatic evaluation of SPT dynamic measurement records. The program reads up to 172 records saved by a Pile Driving Analyzer (PDA). It then performs a data check and, if necessary, certain adjustments. It is advisable to automatically check the records by double integrating the acceleration record. Under certain circumstances (*e.g.*, a high lateral impact components, an error may be introduced into acceleration measurement and therefore into the calculated velocity. Then either a correction must be made or the record rejected. Next, the maximum energy transferred to the drill string top (sensor location) is calculated from each record, and its average computed over each 152 mm (6 inch) depth interval. At this point, a standardized corrected N-value, such as N_{60} (Seed et al., 1985), can be calculated. The accepted data is then also converted from measured top force and velocity records to bottom (sampler) force and velocity using a closed form solution based on wave mechanics theory for linearly elastic rods. The final step is the calculation of the ultimate soil resistance from the force against the sampler, and then the determination of the wave equation soil parameters: damping factor and quake.

Data Check

Experience gained from dynamic pile testing shows that a major strength of this method is the measurement of the two related yet independent quantities, force and velocity. As long as no reflections from pile end, pile non-uniformity or soil resistance reach the pile top, these two quantities must be proportional by the rod impedance EA/c. For SPT testing, the rod is basically uniform and both soil resistance and rod bottom reflections reach the pile top only at and after time 2L/c (twice the length of the rod divided by the wave speed in the rod). Thus, during the time period in which most of the energy is transferred from hammer to rod, the two measurements should be essentially proportional. If they are not then the following should be checked:

a) are there any heavy or open connector joints?

b) is the force record consistent and does it return to zero at the end of the record?

c) does the velocity record return to zero at the end of the record?

d) does the integral of the velocity over time equal the final set?

Connectors cause reflections and rod force decreases (or increases) relative to the velocity, when the connectors are not well tightened (or when their mass is greater than the equivalent rod section). The reflection affects the force-velocity proportionality only for a very short time period and it is therefore easy to verify that such short term deviations of proportionality were not the result of inaccurate measurements.

If the proportionality is unacceptable prior to time 2L/c, then a quick check of the consistency of the force record (recommended two strain measurements taken at the same location) will allow for acceptance or rejection of the force measurements. Adjustments of force records are not possible.

If it can be ruled out that the lack of proportionality between force and velocity was caused by reflections from connectors and that the force record was consistent, then any major proportionality difference must have been caused by the acceleration record. A small shift of the acceleration zero line integrated over time (unbalance) can lead to a relatively large difference in the force-velocity proportionality, in a final non-zero velocity, and in a final displacement different from visual final set. A minor adjustment, if necessary, can be performed to salvage some data sets. Dual measurements of force and velocity can allow the data quality to be evaluated. Records which are of inferior quality are easily identified and should be rejected from further use.

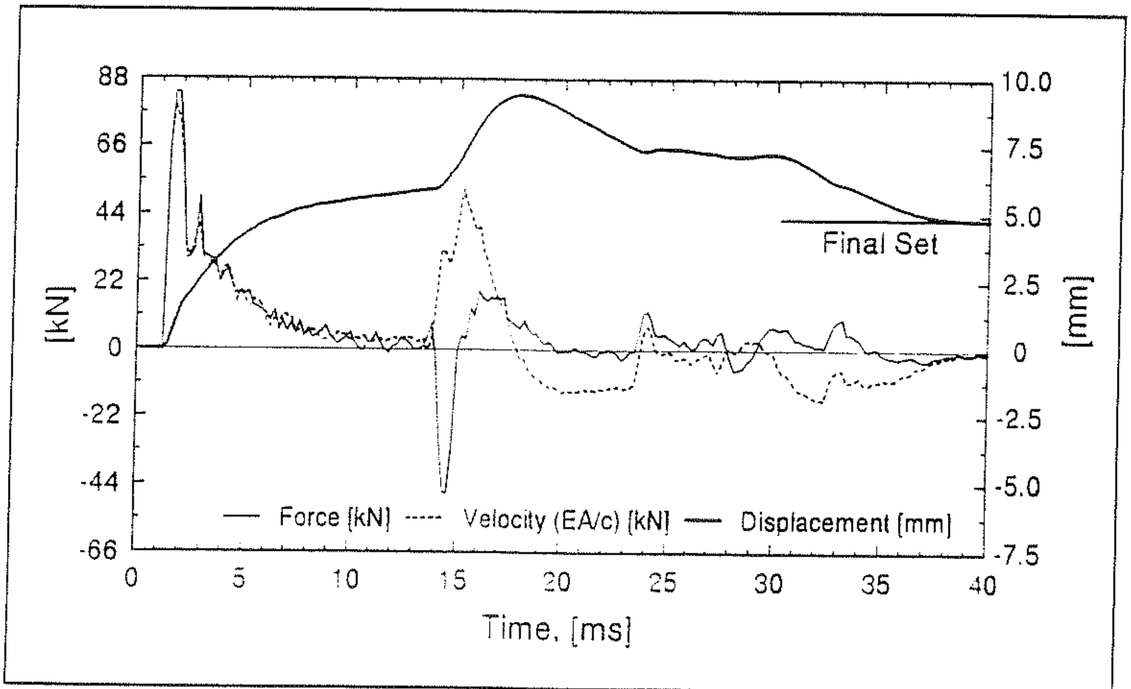

Figure 1: Dynamic SPT Measurement Records: Force, Velocity, and Displacement

For an automatic evaluation of SPT dynamic measurements, data checking is always essential. Figure 1 shows a typical dynamic SPT records: a measured force, an adjusted velocity, and a computed displacement (from the integral of velocity). A computed displacement equal to the observed final set does not necessarily apply to the records of all blows within the same group of blows (required to advance the sampler 152 mm or 6 inches). For example, if the set varies from blow to blow and the blow count therefore is not reliable for final set calculations, then adjustments to meet final set condition should not be made.

Calculation of Bottom Force and Motion

The force at the sampler is equal to the soil resistance, R_{toe}, and can be calculated in closed form from the top measurements of force, $F_m(t)$, and velocity, $\dot{u}_m(t)$, as follows.

$$R_{toe}(t+L/c) = \tfrac{1}{2}[F_m(t) + Z\dot{u}_m(t)] + \tfrac{1}{2}[F_m(t+2L/c) - Z\dot{u}_m(t+2L/c)] \quad (1a)$$

With force, $F_m(t)$ and velocity $\dot{u}_m(t)$ measured near the rod top and with only the existence of toe resistance, $R_{toe}(t)$, the toe velocity can then be calculated with a closed form solution simply as follows.

$$\dot{u}_{toe}(t+L/c) = [F_m(t) + Z\dot{u}_m(t) - R_{toe}(t + L/c)]/Z \quad (1b)$$

where Z (= EA/c) is the impedance of the rod. The toe velocity can be integrated or differentiated to obtain the toe displacement, u(t), or acceleration, $\ddot{u}(t)$, respectively.

Wave Equation Soil Constants

An approach to determine wave equation soil constants for use in wave equation analysis, from the calculated toe force and motion, has been developed under sponsorship by the Federal Highway Administration (Rausche et al. 1996). The basic approach was further refined as described in following discussion. It deals with the calculation of constants for the usual Smith (1960) soil model used in the GRLWEAP program (GRL and Associates, Inc., 1995) and schematically depicted in Figure 2. The total toe resistance, R_{toe}, can be represented analytically as the sum of three separate forces

$$R_{toe}(t) = R_a(t) + R_d(t) + R_s(t) \quad (2)$$

where R_d is the dynamic or velocity dependent resistance, R_s is the static or displacement dependent resistance, and R_a is the inertia or acceleration dependent resistance. Actually, the inertia resistance is only included in the Smith model if it is made a part of a non-uniform pile model. In the case of SPT, it was found convenient to model this resistance as an external force. This acceleration dependent component can be simply written as

$$R_a(t) = m\,\ddot{u}(t) \quad (3)$$

where m may be the additional mass of the sampler, and the soil inside and outside of the sampler that moves with the sampler. According to Smith, the dynamic resistance component is given by

$$R_d(t) = J\,\dot{u}(t)\,R_s(t) \tag{4}$$

where J is the Smith damping constant. As long as the movement of the sampler is downward (loading phase), the static resistance can be written as

$$R_s(t) = (R_u/q)u(t) \;\ldots\; \text{for } u(t) < q \tag{5a}$$

$$R_s(t) = R_u \;\ldots\ldots\ldots\; \text{for } u(t) \geq q \tag{5b}$$

where R_u is the ultimate static capacity, q is the quake, *i.e.*, that sampler displacement at which the static resistance reaches ultimate. With these expressions, and $R_{toe}(t)$, $\ddot{u}(t)$, $\dot{u}(t)$, and u(t) at the toe calculated from top measurements and wave mechanics, the wave equation soil constants R_u, q, J, and m for the toe can be determined.

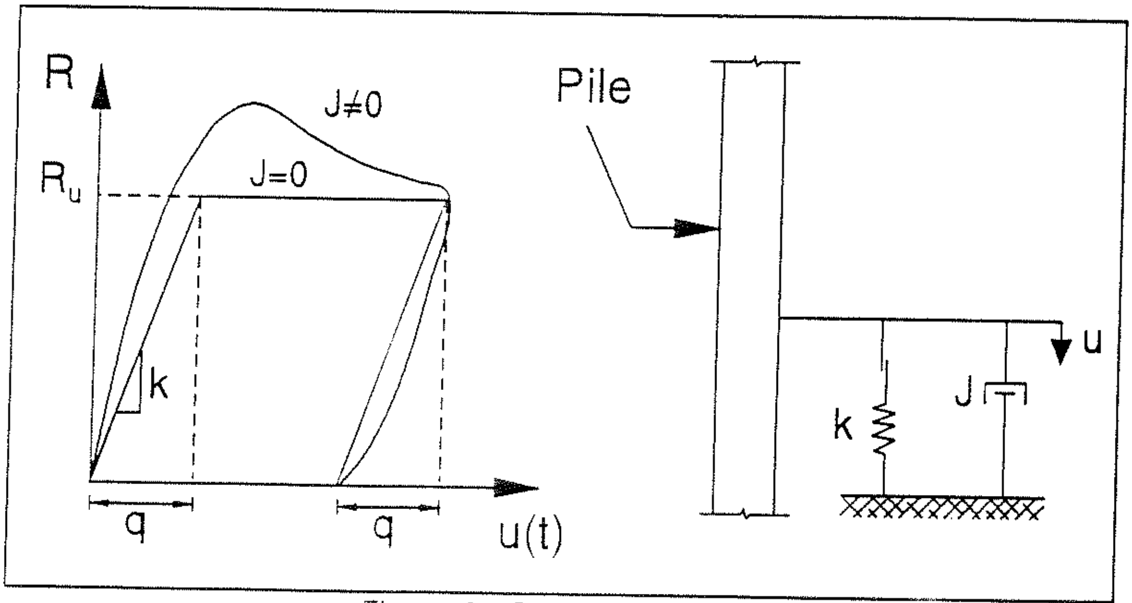

Figure 2: Smith Soil Model

a) Quake, q:

As shown in Figure 2, the Smith soil model assumes that the unloading quake equals the loading quake. Any residual stresses, acting at the sampler after a previous blow, are considered zero. Under these assumptions, the quake can be calculated as the difference between the maximum displacement and the final set. Occasionally, records with very low blow counts are too short for final zero velocity and final set adjustments. In these cases, the quake cannot be determined from the record as proposed above. Instead, the GRLWEAP recommendation (GRL and Associates, Inc., 1995) of d/120 (d is the diameter of pile or sampler) is then applied which leads to 0.42 mm (0.016 inches) in the case of the Split spoon sampler.

b) Ultimate Resistance, R_u:

At the time when the sampler velocity, $\dot{u}(t)$, approaches zero, the acceleration generally is also small (Time A in Figure 3) and therefore, both R_a and R_d vanish. This leaves the static resistance, R_s, as the only unknown in Equation 2. Furthermore, when the velocity reaches zero the displacement is maximum. If the maximum displacement is greater than the quake, the ultimate resistance has been activated and $R_u=R_s=R_{toe}$. If the maximum displacement is less than the quake, R_{toe} would represent the maximum mobilized static resistance which is less than the ultimate resistance.

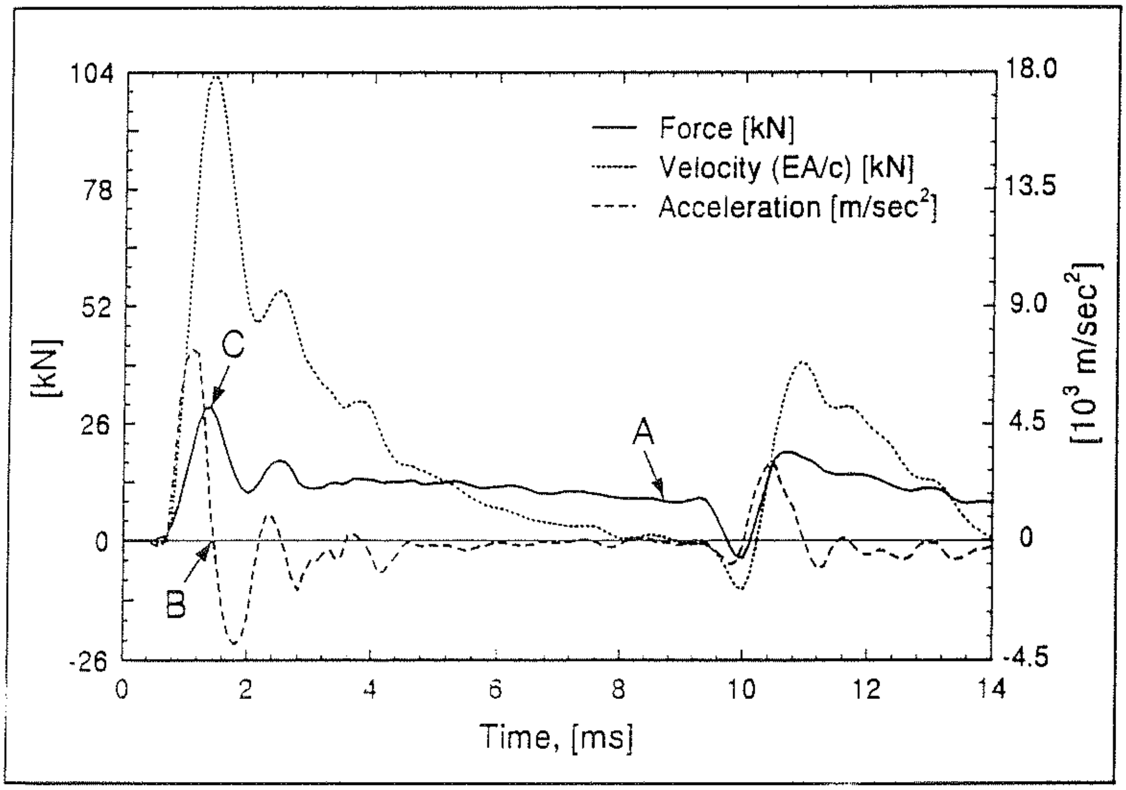

Figure 3: Force, velocity and acceleration at the sampler as calculated from the top measured force and velocity

c) Damping Constant, J:

Damping is calculated at a time when the acceleration becomes zero the first time after impact, *i.e.*, when the velocity reaches a maximum and the inertia force is zero. Thus, at that time (Time B in Figure 3) the damping factor can be calculated with R_s known from $u(t_B)$, q, and R_u.

d) Mass, m:

The mass effect on R_{tpe} is highest at the time of maximum force at impact (Time C in Figure 3) and then quickly decreases to negligible values. The mass, m, is therefore calculated from the force at time C minus the damping and static resistance components at the same time.

Examples

Two examples obtained from different sites are presented showing the results of dynamic SPT measurements using the calculation methods presented above. These measurements were obtained during the course of a research project with a much wider scope than reported in this paper. In particular, a so-called Modified SPT procedure was developed which included the design of static and dynamic test equipment. The Modified SPT included static uplift and torque tests on the sampler equipped drill rod, and compression on a special tip attached to the drill rod end. Such static tests were performed both before and after dynamic driving and testing of the SPT rod, sometimes following certain waiting periods for an assessment of soil setup effects. Complete results and extensive data have been reported by Rausche et al. (1996) and two of the cases reported there are presented here.

Example 1: Results from Modified SPT sampler driving in cohesive soils

Tables 1 and 2 show the results of a test performed at a depth of 19.8 m (65 ft) in a clayey sand (SC, Liquid Limit=19.8, and Plasticity Index=4.4) soil with a blow count of 115 blows per m (N-value of 35 blows per foot). In addition to the normal SPT, after the sampler had been driven the standard 457 mm (18 inches), it was left in the ground for 25 minutes and then restruck again with three 610 mm (24 inches) hammer drops. The equivalent blow count calculated from the set of these three blows was 157 blows/m (48 blows/ft). Two hours later, the sampler was statically uplift tested and then finally again restruck with three 610 mm (24 inches) hammer drops which resulted in an equivalent blow count of 105 blows/m (32 blows/ft).

In this example, nine blows were analyzed: the first three blows were from the end of regular SPT sampler driving, the second set of three blows were taken after the 25 minute wait, and the last three blows were taken after the static uplift test was performed. Table 1 lists the rod wave speed which was calculated from rod length and the wave travel time apparent in the record. This wave speed is slightly lower than for continuous steel rods (normally 5,123 m/s or 16,810 ft/s) because of the presence of partially open connectors. Furthermore, the transferred energy was calculated as the time

Table 1: Summary of the dynamic SPT (with sampler) measurement results and correction factor for the cohesive soil site						
Blow No.	Calculated Wave Speed [m/s]	Acceleration Shift		Blow Count		Transfer Efficiency [%]
		First [%]	Second [%]	Measured [blows/m]	Computed [blows/m]	
1	4952	0.8	.005	115	91	42
2	4894	1.3	.006	115	98	40
3	4894	1.0	.007	115	102	40
Avg	4913			115	97	41
4	4952	2.2	.005	157	178	38
5	4952	0.9	.003	157	168	38
6	4952	1.4	.009	157	128	38
Avg	4952			157	158	38
7	4952	1.4	.008	105	113	40
8	4894	0.3	.003	105	89	40
9	4952	0.5	.005	105	109	40
Avg	4933			105	104	40

Table 2: Summary of soil resistance and wave equation constants for the cohesive soil site dynamically obtained from sampler driving				
Blow No.	"Static" Soil Resistance [kN]	Quake [mm]	Damping [s/m]	Mass [kg]
1	11.6	0.42	0.23	0.16
2	11.2	0.42	0.30	0.14
3	11.6	0.42	0.30	0.15
Avg	11.4	0.42	0.28	0.15
4	13.1	1.78	0.23	0.18
5	11.9	1.52	0.30	0.18
6	12.2	0.42	0.26	0.18
Avg	12.4	1.24	0.26	0.18
7	9.6	0.79	0.36	0.16
8	10.1	0.42	0.36	0.15
9	10.2	0.46	0.30	0.17
Avg	10.0	0.56	0.34	0.16

integral of the product of force and velocity; dividing this "measured energy" by the theoretically available hammer energy (the 64 kg or 140 lb ram dropping 762 mm or 30 inches has an energy of 0.475 kJ or 0.350 kip-ft) yielded the transfer efficiency. The table also includes the acceleration shifts (relative to maximum acceleration), which were applied to achieve proportionality and final zero velocity. Since the final set was not forced as an adjustment, a final set or blow count could be calculated and compared with the measured (equivalent) blow count. For the two sets of restrike blows, the agreement between computed and measured blow counts is very good. For the first set of three blows from end of driving, the agreement is not as good (underpredicting by approximately 16%) probably because the calculated blow count was only based on three blows while the measured one represented 18 blows.

Table 2 shows calculated shaft soil resistances and wave equation soil constants. The soil resistance may be a combination of the resistance at the sampler's shaft and toe . If the shaft resistance is known from a static uplift or a torque test then the toe resistance can be calculated from the total soil resistance.

The soil resistance results (Table 2) show for the second set of three blows a high resistance of 13.1 kN (3.0 kips) which indicates a setup gain of approximately 1.5 kN (0.3 kips) compared to the end of driving resistance (Blow No. 3). The third set of three blows starts with a lower capacity of only 9.6 kN (2.2 kips) and the difference stems from the lack of the sampler's tip resistance following the uplift test. Note that the static uplift test failed at an ultimate resistance of 9.3 kN (2.1 kips) and a quake value of 0.76 mm (0.03 inch); thus, these static measurements agree well with the dynamically calculated results (9.6 kN or 2.2 kips and 0.79 mm or 0.03 inches, respectively) calculated from the first blow after the uplift test (Blow No. 7).

Considering that the resistance occurring during the third series of blows acted primarily along the sampler shaft, the average calculated shaft damping factor was 0.34 s/m (0.10 s/ft). The average calculated shaft damping factor for this clayey sand was within the GRLWEAP recommended range of 0.16 and 0.66 s/m (0.05 and 0.2 s/ft) for cohesionless and cohesive soil, respectively. The calculated mass approximately equals the soil mass inside the sampler.

Example 2: Results from Modified SPT oversized tip in a cohesionless soil

The second example was taken from a special test performed at a depth of 21.6 m (71 ft) in a silty sand soil (SM). In this case, after drilling to the desired testing depth, a special oversized tip was mounted at the end of the drill string, and then reinserted in the drill hole. This tip had a flat bottom and a diameter of 64 mm (2.5 inches) which exceeded the rod diameter (45 mm;

Table 3: Summary of the dynamic SPT (flat bottom tip) measurement results and correction factor for the cohesionless soil site						
Blow No.	Calculated Wave Speed [m/s]	Acceleration Shift		Blow Count		Transfer Efficiency [%]
		First [%]	Second [%]	Measured [blows/m]	Computed [blows/m]	
1	4687	1.1	0	59	108	48
2	4712	0.0	0	59	72	51
3	4712	0.4	0	59	72	48
4	4409	0.8	0	59	75	46
5	4737	0.8	0	59	59	48
6	4763	0.7	0	59	49	48
7	4387	2.5	0	59	49	48
8	4763	1.3	0	59	49	55
Avg	4646			59	67	49

Table 4: Summary of soil resistance and wave equation constants for the cohesionless soil site from flat end bottom tip driving				
Blow No.	Soil Resistance [kN]	Quake [mm]	Damping [s/m]	Mass [kg]
1	10.2	1.91	0.23	0.12
2	9.8	0.53	0.20	0.11
3	8.4	0.46	0.26	0.11
4	6.3	0.42	0.65	0.15
5	6.8	0.28	0.30	0.10
6	6.8	0.42	0.20	0.10
7	2.6	0.42	2.70	0.15
8	6.5	0.42	0.20	0.09
Avg	8.1	0.67	0.23	0.11

1.75 inches). Using the SPT hammer, the rod with the oversized tip was then driven and tested for a penetration of 152 mm (6 inches) with an equivalent blow count of 59 blows/m or 18 blows/ft (actual blow count was 9 blows for 152 mm or 6 inches). A static compression test followed the dynamic test. Very probably the oversize tip precluded any shaft resistance during both dynamic and static testing. The static compression test indicated an ultimate resistance of 7.0 kN (1.6 kips) and a quake of 2.0 mm (0.08 inches).

The dynamic SPT monitoring apparently missed one blow during recording and, therefore, Table 3 summarizes the dynamic results of only 8 blows. Blow numbers 4 and 7 indicated an unusually low wave speed and therefore those results should be ignored. Excluding blow numbers 4 and 7, the dynamic tests (Table 4) indicated static resistance values between 6.5 and 10.2 kN (1.5 and 2.3 kips). Since the static compression test was performed after the dynamic test, the average soil resistance of the last four blows should be used for comparison with the ultimate resistance determined from the static test. The average soil resistance for the last 4 blows (excluding Blow No. 7 with questionable wave speed) is 6.7 kN which agrees well with the static compression test results. Quakes ranged between 0.28 and 0.53 mm (0.01 and 0.02 inches) if the first value of 1.9 mm (0.07 inches) is ignored. The big difference between the calculated and measured quake (from static test) occurred because the program actually calculates the unloading quake, in this case 0.89 mm (0.035 inches). The average damping constant for the six good blows in this silty sand was 0.33 s/m (0.10 s/ft) which is again between the normal GRLWEAP recommended range of 0.16 and 0.66 s/m (0.05 and 0.2 s/ft) for cohesionless and cohesive soil, respectively. Again all calculated results from dynamic testing seem to be very reasonable.

Conclusions

Measurements of force and velocity on SPT installations can be made reliably. These measurements can be used to calculate energy delivered into the rod to calculate the N_{60} value. A method for calculating wave equation soil constants from dynamic measurements on SPT sampler driving has been presented. The method includes the measurement of force and velocity at the drill string top, an automated data check, and adjustment (if necessary), and a simple automated wave equation parameter determination.

In general, agreement of dynamically with statically determined parameters static resistance and quake were quite good. The calculated damping parameters seem reasonable compared with normal past recommendations. While the example cases presented show good agreement with static tests on the SPT rod and past experience, the data base is currently limited and further comparisons of static and dynamic tests on the SPT should be made. The validity of these quantities determined on SPT samplers as they may then be applied to full scale piles still has to be checked.

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